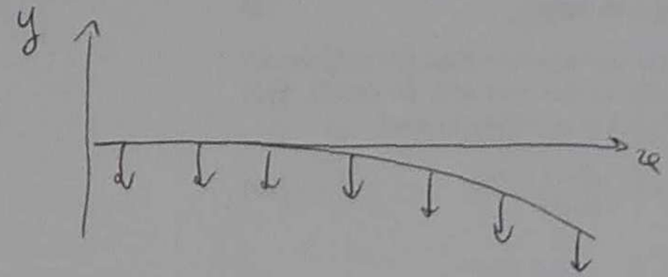
Partiel 2018-2019 - Correction

1- Flexion d'un plongeoir



$$EI\,\frac{\partial^4 y}{\partial x^4} = F_L$$

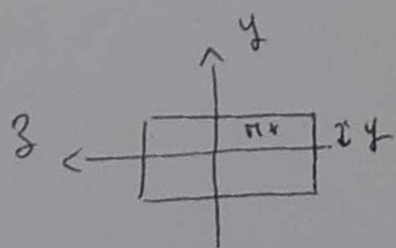
1- Masse linéique $m_L = \dfrac{m}{L} = \rho b e$

$$P_L = m_L g = \rho b e g$$

2- E module d'Young

I moment quadratique

3- $$I = \iint y^2\,dy\,dz$$



$$= b\left[\frac{y^3}{3}\right]_{-e/2}^{e/2}$$

$$= \frac{be^3}{12}$$

4- en $x=0$: $y=0$, $\dfrac{\partial y}{\partial x} = 0$

en $x=L$: $\dfrac{\partial^2 y}{\partial x^2} = 0$, $\dfrac{\partial^3 y}{\partial x^3} = 0$

5- $$\frac{\partial^4 y}{\partial x^4} = \frac{\rho b e g}{EI} = 12\,\frac{\rho b e g}{E b e^3} = 12\,\frac{\rho g}{E e^2}$$

$$\frac{\partial^3 y}{\partial x^3} = \frac{12\rho g}{E\cancel{e}^2} x + A$$

$$= \frac{12\rho g}{Ee^2}(x - L)$$

$$\frac{\partial^2 y}{\partial x^3} = \frac{12\rho g}{Ee^2}\left(\frac{x^2}{2} - Lx\right) + B$$

$$= \frac{12\rho g}{Ee^2}\left(\frac{x^2}{2} - Lx + \frac{L^2}{2}\right)$$

$$\frac{\partial y}{\partial x} = \frac{12\rho g}{Ee^2}\left(\frac{x^3}{6} - L\frac{x^2}{2} + \frac{L^2}{2}x\right) + \underbrace{C}_{=0}$$

$$y(x) = \frac{12\rho g}{Ee^2}\left(\frac{x^4}{24} - L\frac{x^3}{6} + \frac{L^2 x^2}{4}\right) + D = 0$$

$$= \frac{\rho g x^2}{Ee^2}\left(\frac{x^2}{2} - 2Lx + 3L^2\right)$$

6 — $$y_M = \frac{\rho g L^4}{Ee^2}\left(\frac{1}{2} - 2 + 3\right)$$

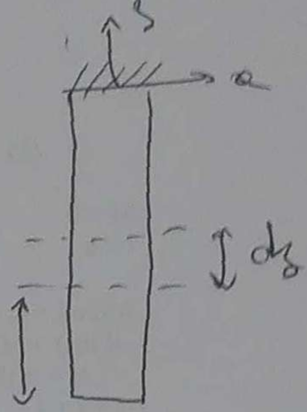
$$= \frac{3\rho g L^4}{2Ee^2}$$

$$= \frac{\cancel{3} \times \cancel{2}.10^3 \times 10 \times 3^{\cancel{4}\,3}}{\cancel{2} \times 2.10^9 \times (\cancel{3}.10^{-2})^2}$$

$$= \frac{27}{2}\,\frac{10^4}{10^9 \times 10^{-4}} = 1{,}4\ m$$

$\Rightarrow$ c'est enorme. Ce plongeoir est trop long.

2 - Allongement d'une poutre sous son propre poids



1 - $\sigma = \frac{\rho g S (L+z)}{S} = \rho g (L+z) = \sigma_0$

2 - $\underline{\underline{\sigma}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \rho g (L+z) \end{bmatrix}$

$\underline{\underline{\varepsilon}} = \begin{bmatrix} -\frac{\nu}{E}\sigma_0 & 0 & 0 \\ 0 & -\frac{\nu}{E}\sigma_0 & 0 \\ 0 & 0 & \frac{\sigma_0}{E} \end{bmatrix}$

3 - $dl = \frac{\sigma_0}{E} dz$

$\Delta L = \int dl = \int_{-L}^{0} \frac{\sigma_0}{E} dz = \frac{\rho g}{E}\left(L^2 - \frac{L^2}{2}\right)$

$= \frac{1}{2}\frac{\rho g L^2}{E}$

$\varepsilon_t = \frac{1}{2}\frac{\rho g}{E} L$

4 - $L = \frac{2\varepsilon_t E}{\rho g} = \frac{\cancel{2} \times 10^{-2} \times 2 \cdot 10^{8}}{\cancel{2} \cdot 10^{4}} = 200 \text{ m}.$

3 - Nanoparticules enrobées

1 - interaction dipôle/dipôle, attractives

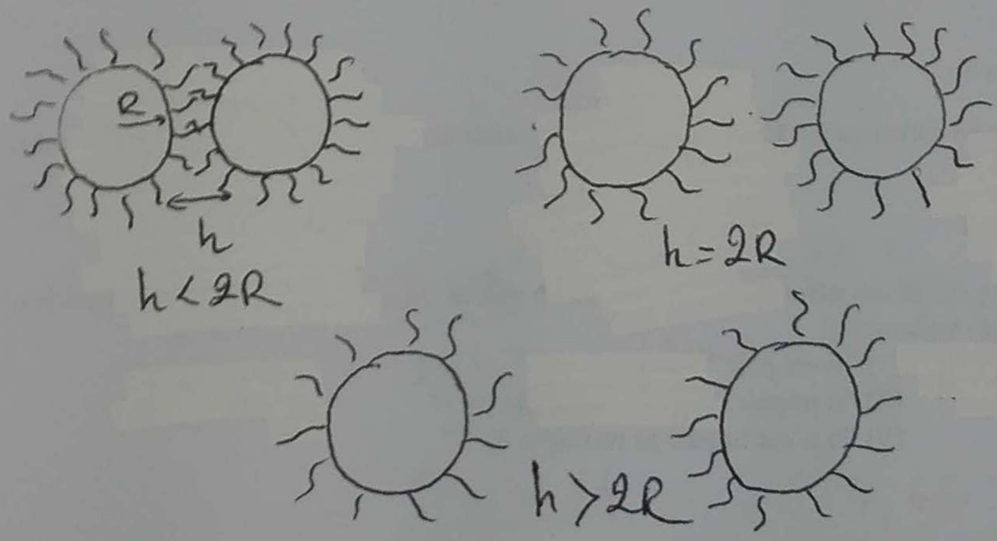
$U \sim \frac{1}{r^6}$

$$U_{vdW} = -\frac{A}{12}\frac{R}{h}$$

2 - $[U]$ = energie donc $[A]$ aussi

3 - entropie

4 -



5 -

$$U = -\frac{A}{12}\frac{R}{h} + kT\left(\frac{2\xi}{h}\right)^{10}$$

$$\frac{\partial U}{\partial h} = \frac{A}{12}\frac{R}{h_0^2} - 10kT\frac{(2\xi)^{10}}{h_0^{11}} = 0$$

Donc

$$\frac{A}{12}R = 10kT\frac{(2\xi)^{10}}{h_0^9}$$

$$h_0 = \left(\frac{120kT(2\xi)^{10}}{AR}\right)^{1/9}$$

6) $$U_0 = -\frac{A}{12}\frac{R}{h_0} + kT\left(\frac{2\xi}{h_0}\right)^{10}$$

$$= -\frac{A}{12}\frac{R}{h_0} + \frac{\cancel{kT}}{h_0}\frac{\cancel{(2\xi)^{10}}\,AR}{120\,\cancel{kT}\,\cancel{(2\xi)^{10}}}$$

$$= \frac{AR}{120 h_0}(1-10)$$

$$= -\frac{9AR}{120 h_0}$$

7) $$U_0 = -kT$$

$$\frac{h_0}{R} = \frac{9A}{120\,kT} = \frac{9\times 4{,}11 . 10^{-19}}{120\times 1{,}38 . 10^{-23}\times 300}$$

$$= \frac{9\times 4{,}11}{1{,}20\times 1{,}38\times 3} = 7{,}45$$

9 - $$h_0 = 7{,}45\,R = 2\xi = 2a\sqrt{N}$$

$$N = \left(\frac{7{,}45\,R}{2a}\right)^2 = \left(\frac{7{,}45\times 10^{-8}}{2\times 10^{-9}}\right)^2$$

$$= 37^2 \sim 1500$$

Question 8

$$\frac{h_0}{R} = \frac{9A}{120\,kT}$$

$$u(h_0) = -\frac{AR}{12\,h_0} + kT\left(\frac{2\xi}{h_0}\right)^{10} = \cancel{-kT}$$

$$= -\frac{120\,\cancel{k}T \times \cancel{A}}{12 \times 9\,\cancel{A}} + \cancel{k}T\left(\frac{2\xi}{h_0}\right)^{10}$$

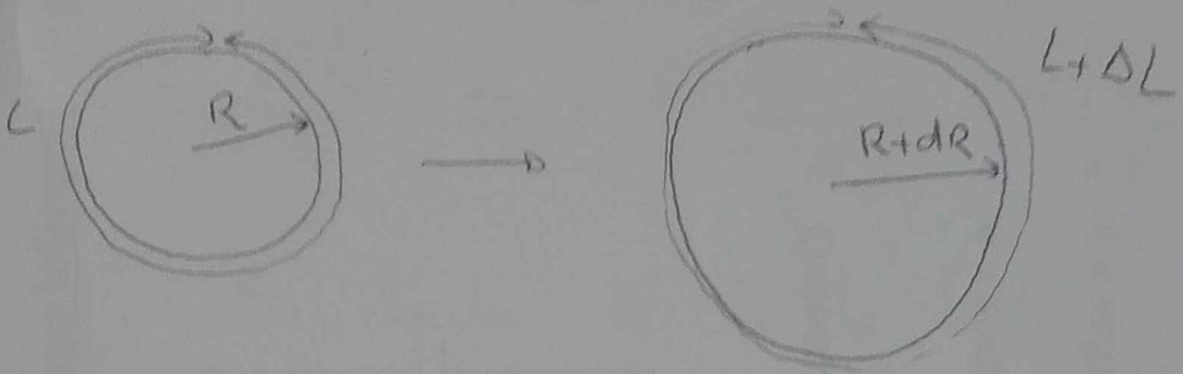
$$\left(\frac{2\xi}{h_0}\right)^{10} = -1 + \frac{10}{9} = \frac{1}{9}$$

$$\frac{2\xi}{h_0} = \sqrt[10]{\frac{1}{9}}$$

$$\frac{2\xi}{R} = \frac{2\xi}{h_0} \times \frac{h_0}{R} = 7{,}45 \times \sqrt[10]{\frac{1}{9}}$$

$$= 8{,}03$$

4 - Le collier de perles



* Rayon du collier de perles R
* Rayon de la tête R_T
* Modules élastiques E, ν
* Contrainte à la rupture σ_r

On agrandit le collier de perles pour l'enfiler

$$R \longrightarrow R + dR = R_T$$

<u>déformation longitudinale</u>

$$\varepsilon = \frac{\Delta L}{L} = \frac{2\pi R_T - 2\pi R}{2\pi R}$$

$$= \frac{R_T - R}{R} = \boxed{\frac{R_T}{R} - 1 = \varepsilon}$$

<u>contrainte correspondante</u> (pour obtenir cette déformation)

$$\sigma = E\,\varepsilon = E\,\frac{R_T - R}{R}$$

Il faut que $\sigma < \sigma_r$. A la limite, on a

$$\sigma_r = E\,\frac{R_T - R}{R} = E\left(\frac{R_T}{R} - 1\right)$$

On cherche R à la rupture.

$$\frac{R_T}{R} = \frac{\sigma_R}{E} + 1$$

$$R = \frac{R_T}{1 + \frac{\sigma_R}{E}}$$

D'où la longueur minimale de fil

$$L = 2\pi R = \frac{2\pi R_T}{1 + \frac{\sigma_R}{E}}$$

On prend $R_T = 15\,\text{cm}$.

$$L = \frac{2\pi \times 0{,}15}{1 + \frac{2}{1}} = \frac{2\pi \times 0{,}15}{3}$$

$$\boxed{L_m \simeq 30\,\text{cm}.}$$