7.03 Laboratory Studies of Mantle Convection

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7.03.1 Introduction

Because laboratory experiments are crucial for exploring new physics and testing theories, they have long played a central role in investigations of thermal convection and mantle dynamics. This chapter is devoted to laboratory experiments considered as tools for understanding the physics that governs mantle dynamics. We shall review both the techniques employed to run well-controlled experiments and acquire quantitative data, and the results obtained.

Investigations of mantle dynamics and of thermal convection have long been closely intertwined. Indeed, the mantle is cooled from above, heated from within by radioactive elements, and heated from below by the core, which loses its heat through the mantle (see Chapter 7.06). The emergence of mantle convection models was dictated by the failure of static, conductive, and/or radiative thermal history models to account for the mantle temperature regime, the Earth energy budget, and the Earth’s lateral surface motions. In the late 1930s, Arthur Holmes, among others, hypothesized that thermal
Convection in the Earth’s mantle provided the necessary force to drive continental motions (Holmes, 1931). Convection, which transports heat by material advection, is the only physical mechanism capable of explaining these observations. The force driving advection is gravity, whereby material lighter than its environment rises while denser material sinks. Such density anomalies can be produced by differences in composition and/or temperature. In the simple configuration of a plane layer (Figure 1), the former will give rise to Rayleigh–Taylor instabilities, while the latter will generate Rayleigh–Bénard instabilities.

Rayleigh–Bénard convection develops when a plane layer of fluid is heated from below and cooled from above (Figure 1). It has been identified as a major feature of the dynamics of the oceans, the atmosphere, and the interior of stars and planets. First identified by Count Rumford (1798), the phenomenon was observed several times in the nineteenth century (e.g., Thomson, 1882). However, it was the carefully controlled and quantitative laboratory experiments of Henri Bénard (1900, 1901) that focused the interest of the scientific community on the problem. Bénard studied the patterns of convection developing in thin layers with a free upper surface. He was interested in the influence of viscosity on the pattern and use several fluids, including spermaceti and paraffin. He determined quantitatively the characteristic length scales of the patterns, the deformation of the interface, and the direction of flow within the fluid. Although we now know that the beautiful hexagonal patterns he observed (Figure 1(c)) were due to the temperature dependence of surface tension (Pearson, 1958), it was these experiments which motivated Lord Rayleigh to apply hydrodynamic stability theory to thermal convection in the absence of surface tension (Rayleigh, 1916). When the thermal convection experiments were carried out correctly, they were found to be very well predicted by Rayleigh’s theory (e.g., Schmidt and Milverton, 1935; Silveston, 1958). For more on the history of these investigations, see Chapter 7.01.

Thermal convection in an isoviscous fluid is characterized by two parameters. The Rayleigh number \( Ra(H, \Delta T) \) compares the driving thermal buoyancy forces to the resisting effects of thermal diffusion and viscous dissipation across the whole system:

\[
Ra(H, \Delta T) = \frac{\alpha g \Delta TH^3}{\kappa \nu} \quad [1]
\]

where \( H \) is the depth of the layer, \( \Delta T \) the temperature difference applied across it, \( g \) the gravitational acceleration, \( \alpha \) the thermal expansivity, \( \kappa \) the thermal diffusivity, and \( \nu = \eta / \rho \) the kinematic viscosity. Convection starts when \( Ra \) exceeds a critical value (Rayleigh, 1916; see Chapters 7.02 and 7.04), and exhibits a sequence of transitions toward chaos as \( Ra \) increases. The second parameter is the Prandtl number, \( Pr \), the ratio of the diffusivity of momentum and that of heat:

\[
Pr = \nu / \kappa \quad [2]
\]
When $Pr >> 1$, the fluid motion stops as soon as the heat source disappears, that is, inertial effects are negligible compared to viscous effects. This is the case for the Earth’s mantle, where $Pr > 10^{13}$.

Numerous theoretical, laboratory, and numerical studies in the last 50 years have been devoted to characterizing thermal convection as a function of $Ra$ and $Pr$. Laboratory experiments have proved especially useful for determining patterns and characterizing the high $Ra$ regime. For high $Pr$ fluids, isoviscous convection has been studied for $Ra$ up to $10^9$, which includes the range of values estimated for the mantle ($10^6$–$10^8$).

However, mantle dynamics is much more complicated than isoviscous convection, in particular due to the complex rheology of mantle material (Chapter 7.02). For example, the generation of Plate Tectonics and its coexistence with hot spots is not reproduced in isoviscous fluids. Studies have therefore focused on progressively more complicated systems, either taking a global view (e.g., convection with temperature-dependent rheology, or internal heating) or a more local view to study some particular mantle feature (such as plumes or subduction). Analog laboratory experiments have been extensively used because of four advantages: (1) since they let Nature solve the equations, they can explore new phenomena for which such equations do not yet exist. (2) They can also explore ranges of parameters, or geometries, where the equations are too non-linear to be solved analytically or numerically. (3) They are inherently three-dimensional (3-D). (4) They can usually be run in the appropriate range of mantle parameters (which is not the case for the atmosphere or the oceans).

This chapter is organized as follows: Sections 7.03.2 and 7.03.3 are devoted to experimental setups, fluids, measurements, and visualization methods. Mantle dynamics on geological time scales is dominated by ‘fluid’ behavior, so that we can generally use liquids around room temperature, and fluid mechanics techniques. With the development of computer power and lasers, it has become possible in the last years to measure the temperature, velocity, concentration, and deformation fields in experimental tanks. Sections 7.03.4–7.03.7 focus on gravitational instabilities (Rayleigh–Taylor instabilities and Rayleigh–Bénard convection). Sections 7.03.8–7.03.11 describes the laboratory experiments related to more specific mantle features – plumes, mixing, accretion, and subduction – which are also described more fully in other chapters of this volume.

### 7.03.2 Experimental Setups and Fluids

#### 7.03.2.1 Designing an Experiment: Scaling

The goal of any fluid mechanics modeling is to determine ‘scaling laws’, or functional relations that link certain parameters of interest and the various other parameters on which they depend. These scaling laws then make it possible to predict the behavior of similar systems, such as the mantle in geodynamics. Hence, there is no question of building a miniature Earth in the laboratory. A phenomenon has to be selected, and a simplified laboratory model is then constructed, where only a few parameters can vary and in a controlled manner. Each individual experiment aims at describing, through quantitative measurements, the behavior of the system for a given set of control parameters. By varying systematically the values of these parameters, a database is constituted. Scaling laws derived from fundamental physical principles can then be tested against the experimental data.

However, the results of a laboratory experiment will be applicable to other natural systems, such as the mantle, only if the dynamic similarity between the scaled-down system (laboratory, computer) and the natural system is respected. Dynamic similarity can be viewed as a generalization of the concept of geometrical similarity (see Chapter 7.04), and requires the following:

1. Similar boundary conditions (mechanical, thermal, geometry).
2. Similar rheological laws. In other words, the mechanical equation of state that relates differential flow stress to strain rate should differ only in the proportionality constant (e.g., Weijermars and Schmeling, 1986).
3. Similar balances between the different forces or operative physical effects. $Ra$ and $Pr$ reflect such balances.

Dynamic similarity gives birth to a set of dimensionless parameters. When the governing equations are known, they can be nondimensionalized to make the relevant dimensionless parameters appear explicitly in the equations and/or the initial and boundary conditions. When the equations are not known, one may use the Buckingham–II theorem, which is a consequence of the fundamental principle that the validity of a physical law cannot depend on the units in which it is expressed. According to this theorem, if...
a given experiment is described by \(N\)-dimensional parameters of which \(M\) have independent physical dimensions, then the experiment can be completely described by \((N - M)\)-independent nondimensional combinations of the dimensional parameters (see Chapter 7.04).

Besides \(Ra\), \(Pr\), and ratios that characterize the variations of physical properties such as viscosity within the experimental tank, other important dimensionless numbers for geodynamics include:

1. the Reynolds number \(Re\), which compares the advection of momentum by the fluid motion and the viscous diffusion of momentum:

\[
Re = \frac{UH}{\nu}
\]  

where \(U\) is the characteristic velocity in the system. For the mantle, \(Re \approx 10^{19}\).

2. the Peclet number, \(Pe\), when the flow is forced by a boundary velocity \(U\) (such as a plate velocity). \(Pe\) compares the heat transport by advection and the heat transport by conduction:

\[
Pe = \frac{Ud}{\kappa}
\]

where \(d\) is for example the thickness of the plate.

All systems with the same dimensionless parameter (say \(Ra\)) will behave in the same way, irrespective of their size. However, the timescale and/or the distance over which the phenomenon occurs will depend on the system size. This is how convection in the laboratory on a scale of hours can be analogous to convection in the mantle over geological times. Dynamical similarity and scaling analysis are therefore essential to analog laboratory modeling. Their principles and techniques are discussed in Chapter 7.04.

### 7.03.2.2 Experimental Fluids

Except when focusing on lithospheric processes such as accretion or subduction, laboratory experiments usually assume that mantle material flows like a Newtonian fluid, with a linear relation between stress and strain rate. This is especially true for convection experiments, because non-Newtonian fluids are usually more difficult to characterize, and to handle.

In the mantle \(Pr \sim 10^{24}\). This is not possible to obtain in the laboratory, but because the dynamics becomes independent of \(Pr\) for \(Pr > 100\) (see Section 7.03.5), the use of fluids with \(Pr > 100\) is adequate to ensure the dominance of viscous over inertial effects.

Another consideration is the temperature dependence of viscosity. Solid-state creep is thermally activated and therefore mantle material has a highly temperature-dependent viscosity (e.g., see Chapter 2.14). The viscosity ratio across the lithosphere where the temperature difference is the highest reaches \(10^7\). A number of experiments have investigated in detail the influence of a strongly temperature-dependent viscosity on convection. The viscosity of liquids always depends on temperature. The silicone oils have the smallest, with a change by no more than a factor of \(3\) over 50°C. Sugar syrups have a much stronger temperature dependence, with Golden Syrup (from Tate and Lyle) the winner, showing a \(10^6\) viscosity change between 60°C and −20°C (Figure 2).

Silicone oils are available in different grades (with viscosity at 20°C between 10 mPa s and \(10^3\) Pa s), and should be used when negligible temperature-related viscosity variations are required in an experiment. Thickeners in aqueous solutions have also been
used (e.g., Tait and Jaupart, 1989; Davaille, 1999a, 1999b; Namiki, 2003). By adding less than 2wt.% of thickener (ex: Hydroxy-ethylcellulose, trade name Natrosol), the viscosity of water can be multiplied by $10^7$ (Figure 3(a)). Although the resulting fluid is shear-thinning (Figure 3(b)), there is a Newtonian plateau at the low shear rates that typically obtain in convection laboratory experiments.

Well-controlled experiments require a precise knowledge of the physical properties of the fluids used. Thermal conductivity and specific heat do not change much from one batch to another, and one can generally rely on the values given by the manufacturer. But it is advisable to measure density (and thermal expansion if needed) and viscosity prior to any new experiments. Moreover, because of the temperature dependence of viscosity, experiments should preferably be run in a temperature-controlled laboratory. This will also ensure that electronic measurements do not drift with time.

### 7.03.2.3 Experimental Setup for Convection

A thermal convection experiment should minimize heat losses to allow reliable heat flux determination, and should also allow good flow visualization (pattern, thermal structure, etc.). Unfortunately, these two requirements cannot be perfectly met at the same time. We shall therefore present three commonly used experimental setups.

#### 7.03.2.3.1 Horizontal pattern visualization

The convecting fluid is bounded between two glass plates; the bottom is heated from below by hot water flushing along its outer surface, while the top is cooled by cold water flushed along the top, as shown in Figure 4(a) (e.g., Busse and Whitehead, 1971; Richter and Parsons, 1975; Whitehead and Parsons, 1978; White, 1988; Weinstein and Olson, 1990; Weinstein and Christensen, 1991). An alternative is to use as the bottom heater a metal plate whose upper surface is polished to a mirror finish (e.g., Chen and Whitehead, 1968; Heutmaker and Gollub, 1987). The horizontal heat exchangers should be carefully leveled since imperfect horizontal alignment modifies convection patterns and heat transfer (e.g., Namiki and Kurita, 2002; Chilla et al., 2004). Precautions also have to be taken to eliminate lateral inhomogeneities due to the side walls. In Busse and Whitehead’s setup (Figure 4(a)), the area where observations on convection were made was bounded on the sides by walls of 2"-thick polyvinylchloride, whose thermal conductivity is close to that of the working fluid (silicone oil). This provided a working area 80 cm x 80 cm. Outside these walls was another convecting region approximately 20 cm in width so that the temperature gradients on both sides of the walls were similar. Visualization was done using the shadowgraph technique (see Section 7.03.3), whereby collimated light (i.e., with parallel rays) is directed vertically up through the tank (Figure 4(a)). In the convecting fluid, the index of refraction of light depends on temperature such that light rays diverge away from hot regions and converge toward colder regions. The projection of the resulting rays onto a white screen thus show hot zones as dark shadows and cold zones as concentration of brightness.

#### 7.03.2.3.2 Heat flux determination

The dependence of the global heat transport on convection characteristics is one of the fundamental...
questions of Rayleigh–Bénard studies (see Section 7.03.4). The most accurate measurement of the global heat flow is obtained through the measurement of the electric power needed to heat the bottom plate (e.g., Schmidt and Milverton, 1935; Malkus, 1954; Silveston, 1958; Giannandrea and Christensen, 1993; Brown et al., 2005; Funfschilling et al., 2005). It requires special design of the

Figure 4  (a) Experimental setup from Richter and Parsons (1975) adapted from Busse and Whitehead (1971). Thermostated water is flowed in the top and bottom glass assemblages. A mylar sheet can be introduced just underneath the top cold plate and driven by a motor at constant speed. (b) Convection apparatus from Giannandrea and Christensen (1993). 1, convection tank; 2, heating plate; 3, heating foil; 4, guard heaters; 5, thermal insulation; 6, working fluid; 7, oil-filled gap; 8, cooling block; 9, nickel wire; 10, device for vertical displacement of the Ni-wire; 11, thermocouples; 12, air gap; 13, adjusting screw. (c) ‘hybrid’ experimental setup. The insulation windows can be removed for visualization.
experimental tank in order to minimize the heat losses from the heating plate to the bottom (e.g., Figure 4(b)). A second experimental problem is the influence of the side walls on the heat transport in the fluid. This problem is significantly reduced when the experimental tank aspect ratio is large (e.g., Giannandrea and Christensen, 1993), and by choosing a wall of relatively small thermal conductivity (e.g., plexiglas) compared to that of the fluid, plus another insulation in low conductivity material such as polystyrene foam. Even with that, side losses must be estimated, and models have been derived to correct for them (e.g., Ahlers, 2001). A third problem is the effect of the finite conductivity of the top and bottom plates on the heat transport by the fluid (e.g., Chilla et al., 2004; Verzicco, 2004): the less conductive the plates, the more the heat transport in the fluid is diminished, and the effect is a function of $Ra$. A correction has been proposed, which fits well the data for turbulent convection (Verzicco, 2004; Brown et al., 2005). In practice, the best conductor available is copper ($k = 319 \text{ W m}^{-1} \text{K}^{-1}$), followed by aluminum ($k = 161 \text{ W m}^{-1} \text{K}^{-1}$). So good heat flux determination precludes the visualization of the planform because of the metal plates, and also visualization from the side because of the insulation.

### 7.03.2.3 Hybrid solution at high $Pr$ and $Ra$

It is of course necessary to correlate heat flux measurements with the geometry of convection. Moreover, modern visualization techniques can give *in situ* measurements of the velocity, temperature, and concentration fields (see Section 7.03.3), but they require transparent sides. Therefore, a number of studies have used a ‘lighter’ setup (Figure 4(c); e.g., Olson, 1984; Jaupart and Brandeis, 1986; Davaille and Jaupart, 1993; Weeraratne and Manga, 1998). The bottom and top heat exchangers are made of copper or aluminum, and are regulated either by electric heating or by circulation of thermostated water. The tank sides are made of plexiglas or glass. The whole tank is insulated with styrafoam, but the insulation on the sides can be removed for visualization. In this configuration, it is advisable to run the experiments in conditions such that the mean temperature inside the experimental cell is close to ambient temperature, to minimize heat losses.

Last, to obtain high Rayleigh numbers ($10^6$--$10^9$) in high $Pr$ viscous fluids, we need fluids thicknesses between at least 10 and 50 cm. It is then no longer possible to have large aspect ratios. It therefore will always be a possibility that the flow will be influenced by the mechanical boundary conditions (zero velocity) at the side walls. We shall discuss this question in more detail later.

#### 7.03.2.3.4 Moving boundaries and free-slip boundaries

To impose a moving upper boundary (e.g., to simulate plate tectonics), a mylar sheet (Figure 4(a)) is usually introduced just below the upper plate and slowly driven by a step-motor (e.g., Richter and Parsons, 1975; Kincaid et al., 1995, 1996; Jellinek et al., 2003).

Because the mantle is bounded by the liquid core at the bottom, and oceans or atmosphere at the top, its mechanical outer boundary conditions are free slip. Moreover, analytical and numerical models usually are best resolved with free-slip boundary conditions. To obtain the latter in the laboratory, thin layers of a fluid much less viscous (at least a 1000 times) than the working fluid must be introduced between the heat exchangers and the high $Pr$ convecting fluid. For a bottom free-slip condition, thin layers of mercury (e.g., Solomon and Gollub, 1991) or salted water (e.g., Jellinek et al., 2002; Jellinek and Manga, 2002, 2004) may be used; while a thin layer of conductive oil may be used to obtain a free-slip upper surface (e.g., Giannandrea and Christensen, 1993). However, since these layers have a finite conductivity, they will modify the thermal boundary conditions as discussed above.

#### 7.03.2.3.5 Centrifuge

Some experiments have been run in centrifuges (e.g., Ramberg, 1972; Nataf et al., 1984; Weiemars, 1988), where the model is subjected to a centrifugal force able to produce accelerations up to 20000g. This approach decreases the experimental time required and allows the use of very viscous materials (with $Pr > 10^5$) at Rayleigh numbers up to $6 \times 10^5$. A detailed description of the experimental setup is given by Nataf et al., (1984) and Weiemars (1988). However, this technique is heavy to implement. With the apparent confirmation (in part because of the early centrifuge experiments) that convective dynamics becomes independent of $Pr$ when $Pr > 100$, the technique has not been used extensively.
7.03.3 Measurements and Visualization Techniques

7.03.3.1 Patterns

Laboratory experiments have been used extensively to determine convective and/or mixing patterns. One then seeks quantitative information on the morphology, wavelength, and evolution (plume ascent rate, blob stretching, etc.) of a particular feature in the experimental fluid.

7.03.3.1.1 Dye

7.03.3.1.1.(i) One-shot visualization

Patterns and motions in two-fluid experiments such as compositional plumes (Figure 5, Olson and Singer, 1985, Section 7.03.6), Rayleigh–Taylor instabilities (e.g., Whitehead and Luther, 1975; see Section 7.03.4), mixing (e.g., Ottino, 1989; see Section 7.03.9) or two-layer convection (e.g., Olson, 1984; Olson and Kincaid, 1991; Davaille, 1999; see Section 7.03.9) are easily observed by the addition of dye to one of the transparent fluids. Most commonly used dyes are fluoresceine, rhodamine, or supermarket food dye. The latter is very easily visualized through illumination with a white light (Figure 5), while the former are most spectacular using laser sheets (e.g., Tsinober et al., 1983; Fountain et al., 2000). In mixing experiments, the dye can be continuously injected by seringes (e.g., Fountain et al., 2000; Kerr and Mériaux, 2004). In all cases, the dye quantity is so small that it does not change the physical properties of the fluids.

The fluid initially at rest can also be marked by dye streaks that allow subsequent fluid motions to be recorded by their distortion. Griffiths (1986) visualized the motions of initially cold fluid due to the passage of a thermal by injecting (from a seringe) before the run at various heights a number of horizontal lines of the same fluid containing a small concentration of dye (Figure 6(a)). Using very viscous putties, Weijermars (1988, 1989) imprinted black grids on some cross-sections prior to experiments in a centrifuge, which recorded the subsequent deformation (Figure 6(b)).

7.03.3.1.1.(ii) Electrochemical technique

In water and aqueous solutions, an electrochemical technique using thymol blue, a pH indicator, can be employed (Baker, 1966). Thymol blue is either blue or yellow-orange depending upon whether the pH of the solution is greater or less than 8. Approximately, 0.01% by weight of thymol blue is added to the water and the solution is titrated to the end-point with sodium hydroxyde. Then, by drop addition of hydrochloric acid, the solution is made orange in color. When a small DC voltage (~10–20 V) from a dry-cell source is impressed between a pair of electrodes situated within such a fluid, H+ ions are removed from solution at the negative electrode, with a corresponding change in color from orange to blue. The dye thus created is neutrally buoyant and faithfully follows the motion of the fluid. Sparrow et al. (1970) used the bottom plate of the experimental cell as the negative electrode itself, while the positive electrode was a large copper sheet situated adjacent to the fluid but well removed from the bottom plate. They hereby obtained the first photographs of thermals rising from a heated horizontal surface (Figure 7). Haramina and Tilgner (2004) recently used the same method to image coherent structures in boundary

Figure 5  Morphology of starting plumes when the dyed plume material is injected into a transparent medium. (a, b) Purely compositional plumes (a) diapiric plume, more viscous than its surroundings; (b) cavity plume, less viscous than its surroundings. (c) Starting thermal plume: the injected dyed material is hotter and less viscous. (a, b) From Olson P and Singer H (1985). Creeping plumes. Journal of Fluid Mechanics 158: 511–531. (c) From Laudenbach N and Christensen UR (2001). An optical method for measuring temperature in laboratory models of mantle plumes. Geophysical Journal International 145: 528–534.
layers of Rayleigh–Bénard convection at very high Rayleigh numbers.

7.03.3.2 Shadowgraph and schlieren
Since the refractive index of a fluid depends on temperature and composition (e.g., salt or sugar content), convective features in a transparent medium will alter the refractive index distribution, also called schlieren (from the German; sometimes spelled ‘shlieren’ in the literature). From the deformation of the optical wave front, one can therefore deduce information on the refractive index, hence on the temperature or compositional field. The optical methods are divided into two groups: the shadow and schlieren techniques using the deflection of light in the measurement media (e.g., Settles, 2001), and interference methods based on differences of length of the optical paths, that is, on the phase (e.g., Hauf and Grigull 1970). In the following, we shall refer to a ‘schlieren object’, when considering the influence of this object on light deflection, and to a ‘phase object’ when considering its influence on the optical path length.

Both shadow and schlieren techniques are whole-field integrated optical systems that project line-of-sight information onto a viewing screen or camera focal plane. They use light intensity and light ray identification, so that only a good white light source is required. They are more appropriate for two-dimensional (2-D) or axisymmetric phenomena since they integrate the information along the light ray path, but are still qualitatively useful for any phenomenon. Both techniques have a long history: R. Hooke developed a schlieren method as early as 1672 and the first published shadowgram of a turbulent plume over a flame by Marat dates back to 1783. Settles (2001) gives a comprehensive review of these two techniques.

7.03.3.2.1 Shadowgraph
Without the object present in the field of view, the light source illuminates the screen uniformly. With the object present, some rays are refracted, bent, and deflected from their original paths, producing a shadow. The optical inhomogeneities of the object redistribute the screen illuminance. The illuminance level responds to the second spatial derivative or Laplacian of the

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Figure 7 Thermal boundary layer instabilities in water visualized by an electrochemical technique. Adapted from Sparrow EM, Husar RB, and Goldstein RJ (1970). Observations and other characteristics of thermals. Journal of Fluid Mechanics 41: 793–800.
refractive index. Best results are obtained when the cell is illuminated by a (quasi-) parallel source of light (Figures 4(a) and 8(a)). Due to the negative slope of the temperature dependence of the refractive indices of most liquids, the hot material acts as a diverging lens and appears dark (Figure 8(b)), while the cold material acts as a focusing lens and appears light (Figure 8(c)). Shadowgraph techniques are easy (and cheap!) to implement and allow the visualization of large objects. Since they image strong temperature gradients well, they have been used to determine convective patterns and measure their characteristic wavelength (e.g., Chen and Whitehead, 1968; Heutmaker and Gollub, 1987; see Section 7.03.5), measure thermal boundary layer thicknesses (Olson et al., 1988), or follow the evolution of laminar thermal plumes (e.g., Shlien, 1976; Moses et al., 1993). However, the whole image cannot be interpreted quantitatively in terms of temperature.

7.03.3.1.2.(ii) Schlieren The schlieren image is a conjugated optical image of the schlieren object formed by a lens or a mirror. The method requires a knife-edge or some other sharp cutoff of the refracted light. Figure 9(a) presents the diagram of a simple schlieren system with a knife-edge placed at the focus of the second lens. Adding an object will bend light rays away from their original paths. The refracted rays miss the focus of the optical system. The upward-deflected ray brightens a point on the screen, but the downward-deflected ray hits the knife-edge causing a dark point against a bright background. So a vertical gradient of the refractive index is converted into an amplitude difference. In general, the illuminance level of the schlieren image responds to the first derivative of the refractive index on a direction perpendicular to the knife-edge. Figures 9(b) and 9(c) show the schlieren image of a hot plume obtained with a horizontal and vertical knife edge, respectively. Figure 9(d) shows the schlieren image of the same plume using a circular spatial filter of 0.5 mm diameter. The bright line in the center of the image and contours represent the zones with zero refractive index gradient.

For weak disturbances, the schlieren technique has a much higher sensitivity than the shadowgraph. It has been used to visualize the onset of thermal convection above a heated horizontal plate (Schmidt and Milverton, 1935), 2-D boundary
layers along vertical walls (e.g., Hauf and Grigull, 1970), and shock waves (e.g., Settles, 2001). Another advantage over shadowgraph is the 1:1 image correspondence with the object of study. Moreover, 3-D study of the phase object is also possible by using a multisource system (Hanenkamp and Merzkirch, 2005).

Other optical methods can be derived from schlieren techniques. In the so-called ‘lens-and-grid’ techniques, an array of light/dark stripes is used as a background grid. This simple method, also called background grid distortion, can be used when a large field-of-view is necessary with no need for high sensitivity. The sensitivity can be improved by adding another grid between the focusing lens and the image plane (Figure 10). The Moiré method can be seen as variant of lens-and-grid method with grids on either sides of the schlieren object. Dalziel et al. (2000) contributed to this method by simulating electronically the second grid in the image capture device. Another variant called Moiré shearing interferometry (or Talbot interferometer) was used for mapping phase objects such as candle flames (Lohmann and Silva, 1972).

7.03.3.2 Temperature and Heat Flow Measurements

Since the precise characterization of thermal convection requires the quantitative knowledge of the temperature field, temperature measurements are a major step in data acquisition. Local measurements have long been provided by thermocouples or thermistors embedded within the flow, but with the risk of disturbing it. The last 30 years have seen the development of noninvasive methods. Laser beam scan and interferometry techniques can provide accurate temperature structure of 2-D or axisymmetric features. More recently, the use of thermochromic liquid crystals has allowed for the first time the accurate determination of a fully 3-D, time-dependent temperature field.

7.03.3.2.1 Local temperature measurements

Point temperature measurements can be obtained electronically using the variation with temperature of the electrical resistance of metals (Pt resistance
thermometers, thermistors) or using the thermoelectric effect at the junction between two metals (thermocouples).

Pt-thermometers exploit the increase with temperature of the electrical resistance of platinum. The most widely used sensor is the 100 Ω or 1000 Ω platinum resistance thermometer. They are the most accurate and stable sensors over a long time period. However, they are expensive and do not come in diameters smaller than a few millimeters. So they are generally used to calibrate all the other temperature sensors in a lab, but are not used directly in the experimental cell.

Thermistors are made from certain metal oxides whose resistance decreases with increasing temperature. Their behavior is highly nonlinear, which limits their useful temperature span, and they are less stable than Pt-thermometers. However, they are cheaper, and can be produced in very small designs (0.1 mm) with a fast response and low thermal mass. The measurement of temperature then requires an electric power supply and a voltmeter. Properly calibrated, thermistors can give a precision of 0.002°C over a 10°C range. They are used in high Rayleigh number convection experiments in water and lower viscosity fluids (e.g., Castaing et al., 1989; Niemela et al., 2000; Chilla et al., 2004).

Thermocouples are based on the thermoelectric effect: the junction between two different metals produces a voltage which increases with temperature. In order for this thermal voltage to produce a flow of current, the two metals must also be connected together at the other end so that a closed circuit is formed. With different temperatures at the junctions the voltages generated are different and a current flows. A thermocouple can thus only measure temperature differences between the measurement junction and the reference junction. The latter is at a known temperature, which is nowadays usually that of a commercial electronic ice-point cell. The temperature range of thermocouples is bigger than for thermistors and their behavior is more linear.
However, the voltage produced by the thermoelectric effect is very small. For the commonly used type ‘K’ and type ‘E’, it amounts, respectively, to 40 and 60 μV per degree celsius, so that a six-digit voltmeter will allow temperature measurements with an accuracy of ±0.025°C. Their response time is about 2 s, that is, much shorter than the typical timescale of instabilities in viscous fluids.

The temperature sensors can be introduced permanently in the experimental tank either one by one at different locations (e.g., Sparrow et al., 1970; Davaille and Jaupart, 1993; Weeraratne and Manga, 1998), or on given vertical or horizontal profiles (e.g., Guillou and Jaupart, 1995; Le Bars and Davaille, 2002, 2004a). The latter allows one to follow the temperature profiles through time but the number of sensors in a 2-mm-diameter probe is limited to 14. Alternatively, the sensors can be mounted on a stepping motor and moved vertically to measure the vertical temperature profile (Figure 4(b)). With viscous fluids, a large volume of fluid is carried along each time the probe is moved to a new depth. Therefore, the probe must be kept several minutes at the same height in order for the system to equilibrate, and the vertical temperature profiles are determined from the time-averaged measurements at each depth (e.g., Olson, 1984; Giannandrea and Christensen, 1993; Matsumoto et al., 2006).

It is also possible to measure directly an horizontal average of the temperature field by stretching a set of very thin platinum wires (0.2 mm diameter) horizontally across the experimental cell (Figure 8(c); Jaupart and Brandeis, 1986; Davaille and Jaupart, 1993; Giannandrea and Christensen, 1993). Because the wire resistance varies as a function of temperature, these wires are operated in the same way as thermistors, within a circuit made of a stable precision tension generator and a precision resistance. This setup is very delicate and time consuming to calibrate, but allows one to measure every second the horizontally averaged temperature with a 0.1°C accuracy with a six-digit voltmeter. This method was used to study penetrative convection in constant-viscosity fluids (Jaupart and Brandeis, 1986) and in strongly temperature-dependent viscosity fluids (Davaille and Jaupart, 1993, 1994).

The advantages of local temperature sensors are their high precision and the ability to obtain long time series (high-frequency sampling and/or long run) at low cost. However, the sensors are introduced within the fluid, which imposes a zero velocity condition for the flow on their surface. They therefore always perturb the flow locally. For nonsteady flow and thin probes, this perturbation remains negligible and the probes do not preferentially focus convective features on themselves (e.g., Davaille and Jaupart, 1993). However, for steady flow (like low Rayleigh number convection), upwellings or downwellings can be locked on the temperature probes. In such cases, one should use only removable temperature sensors mounted on step motors, and/or the nonintrusive methods to which we turn now.

### 7.03.3.2.2 Deflection of a light beam

In the cases of 2-D or axisymmetrical refractive index distributions, it is possible to recover the temperature field by scanning the test zone with a laser beam and recording its deflection. Nataf and et al., (1988) and Rasenat et al. (1989) applied this method to determine the type of coupling in two-layer convection at low Rayleigh numbers. Laudenbach and Christensen (2001) described the method in detail for the axisymmetric case (Figure 11) and applied it to thermal plume conduit and solitary waves.

The same treatment can be applied to the lens-and-grid methods described in Section 7.03.3.1.2, since the ray displacement can be calculated from the grid distortion provided that the structure is 2-D or axisymmetrical. The spatial resolution of the method then depends on the spatial frequency of the grid lines.

### 7.03.3.2.3 Interferometry

Compared to shadowgraph and schlieren methods, interferometric methods offer more detailed information about the phase object (e.g., Yang, 1989), since they play with the light path, but they usually require laser light and a good optical control. They have been used for pattern determination (e.g., Nataf et al., 1981, 1984, 1988; Jaupart et al., 1984; Kaminski and Jaupart, 2003) as well as for quantitative measurement of the temperature field (e.g., Gebhart et al., 1970; Shlien and Boxman, 1981).

#### 7.03.3.2.3.(i) Mach–Zender interferometry

The most common two-beam interferometer is the Mach–Zender interferometer (Figure 12(a); Hauf and Grigull, 1970). The phase object is placed in one of the legs of the interferometer formed by two beam-splitter mirrors and two total reflecting mirrors. For a 2-D object, the interference fringes can be interpreted as isothermal lines. It has been used for boundary-layer dynamics along vertical walls or in double-diffusive systems (e.g., Lewis et al., 1982), and...
steady laminar plumes above a horizontal line heat source (Figure 12(b); Gebhart et al., 1970). For a 3-D object, the phase difference between the two optical paths is integrated over the longitudinal length of the object, and the interferogram is not readily interpretable (Merzkirch, 1993). In a cylindrical symmetry the radial temperature distribution can be obtained rather simply by an Abel transformation. If the refractive index has no symmetry, tomography algorithms and reconstruction techniques have to be applied. The Mach–Zender interferometer was used to measure the temperature field of axisymmetric, laminar thermal plumes in liquids (Figure 12(c); e.g., Boxman and Shlien, 1978; Shlien and Boxman, 1979, 1981; Chay and Shlien 1986) and for turbulent mixing of salt solutions (Boxman and Shlien, 1981). More recently, Qi et al. (2006) used the Michelson interferometer, more often used to measure surface displacement (see Section 7.03.3.5), to measure the temperature field of 3-D axisymmetrical flames.

Holographic interferometry was introduced in flow visualization by Heflinger et al. (1966) and Tanner (1966). Holography has opened a new dimension for two-beam interferometry: the reference and test beams can be separated in time rather than space. In a holographic interferometer two consecutive exposures are taken through the field of interest, usually the first exposure without object and the second in the presence of an object of varying temperature. The double exposed plate is developed and placed in the holographic reference beam for reconstruction. The pattern which becomes visible after this reconstruction is equivalent to the one obtained by a Mach–Zender interferometer (Merzkirch, 1993; Hauf and Grigull, 1970). The main advantage of holographic interferometry over ‘classical’ interferometers is that, since the geometrical path of the signal in the two exposures is identical, the quality of the optical components is not crucial. The eventual optical disturbances and impurities in the test section cancel out.
7.03.2.3.(ii) Differential Interferometry

Differential interferometry (also called shearing interferometry) is a method to measure derivatives of light phase distortions. In practice, this can be done in two ways. One possibility is to send two beams slightly displaced through a phase object and to put them back together on the camera. The best way for doing this is to use two Wollaston prisms and polarizers (Oertel and Bühler, 1978; Nataf et al., 1981). Another possibility is to divide one wave into two identical beams behind the object and to put them on a camera with a small displacement. This can be done for instance by using one Wollaston prism (Sernas and Fletcher, 1970; Small et al., 1972). Another variant is the use of a Mach–Zender interferometer with the phase under investigation placed outside (Pretzel et al., 1993). In this arrangement, using a very compact Mach–Zender interferometer minimized its inherent sensitivity to vibrations. Another simple way of separating into two beams is to use an optical flat plate tilted at a certain angle (Figure 13; Jaupart et al., 1984; Kaminski and Jaupart, 2003). Extinction lines follow constant horizontal temperature gradients. Vertical extinction lines away from the plume correspond to light beams through uniform background.

The main advantage of differential interferometry is its variable sensitivity: carrier fringe orientation and frequency can be chosen separately. It is therefore possible to visualize with the same apparatus the flow configuration in two fluids with very different temperature dependence of the refractive index (e.g., Oertel and Buhler, 1978). Moreover, the separation between the object and the interferometer facilitates the investigation of large and complicated objects. The evaluation of the interferogram can be done by


**Figure 13** Differential interferometry. (a) Setup. (b) Thermal plume in silicone oil V5000.
Fourier analysis, and the result, being the first derivative of the integral phase shift caused by the object, is obtained with high accuracy. For radially symmetrical objects, the spatial distribution of the refractive index can be obtained easily because the Abel inversion formula is reduced to a simple integration, which can be done more reliably than a differentiation. In that respect, differential interferometry results are less noisy than Mach–Zender interferometry, where the spatial distribution of the refractive index is obtained through numerical differentiation of the optical path difference. More details on the digital processing of interferograms can be found in the monograph by Yang (1989).

Speckle interferometry can also enter into this category (Merzkirch, 1995). In double-exposure speckle photography, two speckle patterns are recorded on the same photographic plate. Between the two exposures the scattering plate (ground glass) is shifted to obtain the interferometric fringes. After photographic development, the specklegram is scanned by a laser beam. By measuring the Young’s fringe spacing and orientation, it is possible to measure the two components of the displacement and convert them into deflection angles (resembling for this reason the shadowgraph and schlieren methods). In addition, precise multiprojection speckle photography allow the reconstruction of a 3-D temperature field using computer tomography (Asseban et al., 2000). Among the most important advantages of speckle photography are its spatial resolution (about 0.2 mm) and the possibility to collect a great amount of experimental information from a single specklegram. Large amounts of data can be processed and analyzed statistically. The potential of analyzing spatial characteristics of turbulent flows was demonstrated (Merzkirch et al., 1998).

7.03.3.2.4 Isotherms: thermochromic liquid crystals
The use of thermochromic liquid crystals (TLCs) allows one to visualize the temperature field on a 2-D-plane in the fluid flow without perturbing it (Rhee et al., 1984; Dabiri and Gharib, 1991; Willert and Gharib, 1991). Liquid crystals are mesomorphic phases which present peculiar properties due to the presence of some degree of anisotropy (Chandrasekhar, 1977). One particular class of these mesophases, chiral nematics (cholesterics), have a structure that undergoes an helical distortion, and because of their periodic structure, they generate Bragg reflections at optical wavelengths. The pitch of the wavelength of the Bragg-reflected light depends on the temperature $T$ (de Gennes and Prost, 1993). Thus, the color of the material can change drastically over a temperature interval of a few degrees. When the liquid crystals are illuminated by white light, their color changes with increasing temperature from colorless to red at low temperatures, passes through green and blue to violet, and turns colorless again at high temperatures. It was first used as paint on a surface to determine convective patterns qualitatively (e.g., Chen and Chen, 1989; Lithgow-Bertelloni et al., 2001).

Subsequently, it became possible to mix the TLC slurry directly within the fluid and to illuminate the tank cell on cross-sections (Figure 14(a)). One of the main applications of this technique has been the study of aqueous turbulent flows (Solomon and Gollub, 1990, 1991; Gluckman et al., 1993; Moses et al., 1993; Dabiri and Gharib, 1996; Park et al., 2000; Pottebaum and Gharib, 2004). The use of this method to measure the temperature field quantitatively requires a high-precision color CCD camera and a very precise calibration of the color of the liquid crystals particles against the true temperature. Moreover, the total temperature range accessible with one particular TLC slurry is usually around 2–3°C. This technique has lately been further extended to the joint measurement of temperature and velocity in a 3-D field (Kimura et al., 1998; Fujisawa and Funatani, 2000; Giofalo et al., 2003). An uncertainty analysis performed by Fujisawa and Hashizume (2001) gives an error less than 0.1°C, or 5% of the total imposed temperature difference, for a calibration method based on a hue-saturation-intensity approach (Fujisawa and Adrian, 1999).

Convection in conditions analogous to those of the mantle involves viscous fluids (see Section 7.03.2) and typical temperature heterogeneities $\sim$10–25°C. Therefore, the use of one TLC slurry is not enough to image the whole temperature field. The use of white light and a single TLC was used to determine the influence of mechanical (Namiki and Kurita, 1999, 2001) and thermal boundary heterogeneities (Matsumoto et al., 2006) on thermal convection. The type of TLC was chosen to image well the hot instabilities. This permitted quantification of the convective pattern, but not accurate measurement of the temperature field. Illuminating several TLC slurries by a single laser wavelength plane sheet and recording the images with a high precision black and white CCD camera
Davaille et al. (2006) demonstrated that it was possible to obtain isotherms and local temperature gradients. The laser beam was expanded into a beam using a cylindrical lens. Each slurry brightens over a different temperature subrange and therefore generates a horizontal bright line (Figure 14(b)). Each stripe presents a finite thickness: although each TLC responds at a given wavelength to a precise temperature, the polymeric capsules enclosing them introduce a scatter around this value. On a plot of the intensity as a function of depth, or temperature, each stripe corresponds therefore to a peak whose maximum defines the value of the ‘isotherm’, and whose thickness gives a measure of the local temperature gradient. After calibration, the light intensity maximum gives the ‘isotherm’ temperature with a precision of \( \pm 0.1^\circ\text{C} \), and the half width of the bright lines gives the local temperature gradient. This method was used to study two-layer convection (Le Bars and Davaille, 2002, 2004a, 2004b; Kumagai et al., 2007), plumes arising from a viscous thermal boundary layer (Davaille and Vatteville, 2005) and plumes from a point heat source (Vatteville, 2004). Since viscous fluid motions are slow, it is possible to scan the experimental cell with the laser sheet (by means of an oscillating mirror driven by a galvanometer) to obtain the 3-D structure of the temperature field. This method can be combined with laser-induced fluorescence (LIF) and particle image velocimetry (PIV) to obtain simultaneously the composition (see Section 7.03.3.3) and velocity fields (see Section 7.03.3.4).

### 7.03.3.2.5 Heat flow measurements

The dependence of the global heat transport on convection characteristics is one of the fundamental questions of Rayleigh–Bénard studies (see Section 7.03.5).

The most accurate measurement of the global heat flow is obtained through the measurement of the electric power needed to heat the bottom plate (e.g., Schmidt and Milverton, 1935; Malkus, 1954; Silveston, 1958; Giannandrea and Christensen, 1993; Brown et al., 2005; Funfschilling et al., 2005). With careful design of the experimental tank in order...
to minimize the heat losses (cf. Section 7.03.2.3), late results have reached a precision of 0.1% (e.g., Funfschilling et al., 2005).

The heat flow can also be deduced from the first derivative of the measured temperature profiles, either locally (using temperature probes or isotherms) or horizontally averaged (using platinum wires). This method requires a precise knowledge of thermal conductivity (especially as a function of temperature), and temperature measurements at several different heights within the thermal boundary layers close to the horizontal plates to resolve the steep gradients there. This becomes increasingly difficult as the Rayleigh number increases and the boundary layers become thinner, with the danger of underestimating the heat flux. For Rayleigh numbers up to $10^8$, a precision of 5% has been obtained (e.g., Davaille and Jaupart, 1993).

### 7.03.3 Composition

Measurement of the time evolution of composition is required in studies on entrainment, mixing and two-layer convection. It usually amounts to measuring a tracer dilution. Point-based techniques use either in situ probes (e.g., conductivity), or extraction of samples by suction from various points in the flow field. Tait and Jaupart (1989) used conductivity measurements to study the mushy crystallization of ammonium chloride in viscous solutions. Davaille (1999) studied entrainment processes in two-layer convection by periodic sampling of the fluid layers, measuring the salinity of each sample using the dependence of the refractive index on the salt concentration, and a food dye concentration by a UV absorption technique. However, these point-based techniques have some major drawbacks: the flow may be disturbed by the probes, the number of measurement points is very limited, and extraction techniques can yield only time-averaged concentrations and cannot capture their instantaneous fluctuations.

Concentration profiles averaged over the experimental cell (e.g., Olson, 1984) or over one of its dimension (e.g., Solomon and Gollub, 1988a, 1988b, 1991) can be deduced from the optical absorption by the cell of an extended white light beam (e.g., Olson 1984), or a laser beam (e.g., Solomon and Gollub, 1988a, 1988b, 1991).

The advent of LIF techniques in the 1970s enabled simultaneous capture of the entire instantaneous tracer (fluorescent dye) concentration field over a planar sampling area (laser sheet), with a experimental set up similar to Figure 14. LIF is a nonintrusive technique which has been applied to turbulent jet flows (e.g., Villermaux and Innocenti, 1999), laminar mixing (e.g., Ottino et al., 1988; Fountain et al., 2000), and two-layer convection (e.g., Kumagai et al., 2007). Since fluoresceine dye shifts the laser light frequency, it is possible to use LIF and TLCs to measure composition and temperature simultaneously, by recording the images with two different filters, one for the laser frequency and one for the fluoresceine (Figure 15; Kumagai et al., 2007). To obtain a 3-D field, the laser beam can be swept through the flow at high speed and images captured with a synchronized camera.

![Figure 15](image-url) Interaction of a thermal plume with a density stratification. The denser layer has been dyed with fluoresceine. Two-dimensional visualization when the tank is illuminated with a laser sheet (532nm): (a) isotherms (26.3, 32.5, and 38.9 °C) and (b) compositional fields (LIF method). These two images were taken by a 3-CCD camera at almost same time, but using two different optical filters, a band-pass filter for temperature and a cut-off filter for composition. From Kumagai I, Davaille A, and Kurita K (2007). On the fate of mantle thermal plumes at density interface. *Earth and Planetary Science Letters* 254: 180–193.
7.03.3.4 Fluid Motions

7.03.3.4.1 Local measurements

It is easy to follow the displacement of an interface defined by a grid (e.g., Weijermars, 1988), or by abrupt gradients either in refraction index (e.g., visualized by shadowgraph or by interferometry) or in dye concentration. These techniques have been used to measure the rising velocity of thermals (Griffiths, 1986) or plumes (e.g., Olson and Singer, 1985; Griffiths and Campbell, 1990; Moses et al., 1993; Coulliette and Loper, 1995; Kaminski and Jaupart, 2003).

‘Hot wire’ probes can also be used whereby the local flow velocity is measured by sensing the rate of cooling of fine, electrically heated wires. However, their use is usually confined to turbulent or high-speed flows of low viscosity fluids. Moreover, as an intrusive method, it suffers of the same drawbacks as temperature sensors (cf. Section 7.03.3.2.1).

Laser Doppler velocimetry (LDV) allows one to measure continuously in time and at a given position in space up to the three components of the velocity of tracer particles (e.g., Adrian, 1983; Merzkirch, 2000). The method is based on the optical Doppler effect and requires seeding of the working fluid with micron-size particles. Incident light is scattered by the moving particles, and the frequency of the scattered radiation is Doppler-shifted. Since this frequency shift is relatively small, its detection requires the use of monochromatic incident laser light. Maps of the spatial dependence of the velocity field can be obtained by translating the cell apparatus over the LDV system. Solomon and Gollub (1988a, 1988b, 1991) applied this technique to measure the velocity field in steady quasi-2-D Rayleigh–Bénard convection.

7.03.3.4.2 2D and 3D field measurements

One of the oldest techniques for measuring velocities in a fluid is seeding particles (e.g., hollow glass beads, aluminum flakes, and tiny air bubbles) into the fluid and illuminating a cross-section of the flow cell by a thin sheet of light. The foreign particles should be small and have densities as close to that of the fluid as possible, in order to follow passively the local flow.

There are then several ways to describe the flow. The trajectory of a single fluid particle over time defines a ‘pathline’. A ‘streakline’ connects all the fluid elements that have passed through a given point (Figure 14(c)). ‘Streamlines’ are tangential to the flow directions at a given time (Figure 14(e)). In steady flow, pathlines, streaklines, and streamlines are identical, but usually not in time-dependent flow.

The availability of high-power laser sources together with fast digital processors led to the development of sophisticated whole-field velocimetry techniques such as PIV and particle tracking velocimetry (PTV). Both techniques provide a quantification of the velocity field over the entire plane. The optical setup is the same as for the visualization of isotherms (Figure 14(a)). PTV tracks individual particles in subsequent images, while PIV follows a group of particles through statistical correlation of local windows of the image field from two sequential images (Adrian, 1991; Raffel et al., 1998; Merzkirch, 2000). From the known time difference and the measured displacement, the velocity is calculated.

PTV is convenient when the seeding is sparse, and/or near boundaries, since each particle trace is analyzed individually. The latter operation is time consuming. Moreover, as the particles are present at random locations in the fluid, the velocity estimators are found at random locations in the flow, too.

The PIV scheme, on the contrary, requires a high particle density, and calculates the ‘mean’ displacement of particles in a small region of the image (the interrogation window) by cross-correlation of the transparency signal of the same window in two subsequent images. It therefore removes the problem of identifying individual particles, which is often associated with tedious operations and large errors in the detection of particle pairs. Its spatial resolution is uniform (function of the interrogation window size) over the whole image. In geodynamics, this technique has recently been used to study subduction (Kincaid and Griffiths, 2003, 2004) and thermal convection (Davaille and Vatteville, 2005; Kumagai et al., 2007).

Current research aims at combining the resolution of PTV with the fast algorithm of PIV. Funicello et al. (2006) recently developed such a feature tracking (FT) method to map mantle flow during retreating subduction.

PIV has also been used simultaneously with TLCs in two ways. When the temperature difference is small and only one TLC slurry is needed to determine a continuous temperature field, the velocity is measured using the same TLC particles as Lagrangian flow tracers (e.g., Dabiri and Gharib, 1991; Park et al., 2000; Ciofalo et al., 2003). When several TLC slurries are used to visualize isotherms, the fluid can also be seeded uniformly with 10 μm size particles (hollow glass or latex) to calculate the...
PIV (Figure 14; Davaille and Vatteville, 2005). 3-D determination of the two fields is possible by scanning the experimental tank with the laser sheet (e.g., Ciofalo et al., 2003). The velocity field can also be obtained in a volume by stereoscopy, whereby two cameras record the particle displacements from different angles.

7.03.3.4.3 Stress and strain rate

Stress and strain rate can be determined from the displacement and velocity fields measured by the methods just described; indeed, this is the most common method.

However, one can also use optical ‘streaming birefringence’. This effect, already known to Mach (1873) and Maxwell (1873), is the property of certain liquids to become birefringent under the action of shear forces in a flow. Such fluids consist mainly of elongated and deformable molecules (e.g., polymers) or contain elongated, solid, crystal-like particles in solution. At rest, these particles are randomly distributed and the fluid is optically isotropic. The shear forces in a flow cause the particles to align in a preferential direction, which renders the fluid anisotropic. This phenomenon is also well known in solids (and used in photoelasticity), and in the mantle is responsible for seismic anisotropy. Light propagation in such a medium is directionally dependent: an incident light wave separates in the birefringent liquid into two linearly polarized components whose planes of polarization are perpendicular to each other, and which propagate with different phase velocities. Different values of the refraction index are therefore assigned to the two components. They are out of phase when leaving the birefringent liquid, and this difference in optical path can be measured by interferometry.

Although theoretically most fluids should show this effect, the best results have been obtained by using Milling Yellow dye dissolved in aqueous solutions (for its physico-chemical properties; see Swanson and Green (1969) and Pinder and Krishnamurthy (1978)). Milling Yellow solutions can be strongly non-Newtonian, depending on concentration and temperature. One special concentration is particularly interesting, since its flow curve is similar to that of human blood (Schmitz and Merzkirch, 1981).

Interferometric fringes, ‘isochromates’, can then be obtained with a Max–Zender interferometer or with a polariscope (Figure 16). However, a theoretically based flow-optic relation that would allow quantitative determination of the flow from the refraction index distribution does not yet exist. Empirical relations concern only small values of the maximum strain rate, or small Reynolds numbers (creeping flow). For 2-D or axisymmetric situations, a linear relationship between the refractive index differences and the strain rate has been successfully used to determine the flow in pipes of varying cross-sections (e.g., Schmitz and Merzkirch, 1981). Horsmann and Merzkirch (1981) developed a flow-optic relationship which applies to the general 3-D case. More details on the technique can be found in Merzkirch (1989).

7.03.3.5 Surface Displacement

On Earth, hot spots are usually located on top of broad topographic swells (see Chapter 7.09). Experiments on the interaction between a rising buoyant plume and the lithosphere, commonly proposed to explain hot-spot volcanism, therefore involved measurement of the upper surface displacement. Two techniques were used. Olson and Nam (1986) measured the amplitude of the surface topography above the axis of a rising buoyant drop using a high-frequency induction coil proximity probe (Figure 17(a)). They were able to follow the temporal evolution of the surface elevation with a sensitivity of 0.02 mm. Using a Michelson interferometer, Griffiths et al. (1989) could reconstruct from the fringe pattern both the shape and width of the swell, in addition to its height (Figure 17(b)). Fringes in the interferogram are topographic contours with a vertical distance apart of half the laser wavelength (i.e., 0.633 μm for a He–Ne laser). The surface height at any position of the swell is therefore given by the number of fringes.
Whenever light fluid underlies a heavier fluid in a field of gravity, the interface between them is inherently unstable to small perturbations (Figure 1(a)). This is the classic form of the so-called Rayleigh–Taylor instability (hereafter RTI), first studied theoretically by Rayleigh (1883) and later by Taylor (1950) (see Chapter 7.04). RTIs have been used to model a number of geophysical processes, including the formation and distribution of salt domes (e.g., Nettleton, 1934; Selig, 1965; Biot and Ode, 1965; Ribe, 1998), the emplacement of gneissic domes and granitic batholiths (Fletcher, 1972), instability of continental lithosphere beneath mountain belts (Houseman and Molnar, 1997), subduction of oceanic lithosphere (Canright and Morris, 1993), the temporal and spatial periodicity of volcanic activity in a variety of geological settings, namely island arcs (Marsh and Carmichael, 1974; Fedotov, 1975; Marsh, 1979; Kerr and Lister, 1988), continental rifts (Mohr and Wood, 1976; Bonatti, 1985; Ramberg and Sjostrom, 1973), Iceland (Sigurdsson and Sparks, 1978), and mid-ocean ridges (Whitehead et al., 1984; Shouten et al., 1985; Crane, 1985; Whitehead, 1986; Kerr and Lister, 1988), segregation and mixing in the early history of Earth’s core and mantle (Jellinek et al., 1999), and the initiation of instabilities deep in the mantle (Ramberg, 1972; Whitehead and Luther, 1975; Stacey and Loper, 1983; Loper and Eltayeb, 1986; Ribe and de Valpine, 1994; Kelly and Bercovici, 1997; Bercovici and Kelly, 1997).

Figure 17 Measurements of surface displacements above a rising drop. (a) Proximity probe. (b) Side view of the rising drop. (c) Holographic interferometry on the free surface. The center of the interference fringes corresponds to the surface highest point and is centered on the drop vertical axis. (a) Adapted from Olson and Nam (1986). (c) From Griffths RW, Gurnis M, and Eitelberg G (1989) Holographic measurements of surface topography in laboratory models of mantle hotspots. Geophysical Journal 96: 1–19.

7.03.4 Rayleigh–Taylor Instabilities

Laboratory experiments have proved to be powerful tools for studying the development and morphology of RTI. We focus here on the case where the density field is nondiffusing, and the fluids highly viscous, so that the effects of compositional diffusion, inertia, and surface tension, can be neglected. Then, the nature of overturning only depends on the geometry of the boundaries, the viscosities, and densities of the fluids and the layer depths. A large number of experimental studies using a variety of materials have been performed. For the experiments using the most viscous fluids (e.g., silicone putties), the effective gravity was enhanced up to 800g by spinning in a centrifuge (e.g., Ramberg, 1967, 1968; Jackson and Talbot, 1989). Early experiments with putty and other non-Newtonian fluids have been extensively photographed and compared with geological formations (esp. salt domes) by Nettleton (1934, 1943), Parker and McDowell (1955), and Ramberg (1967, 1972). However, there was no intercomparison between these laboratory experiments and theory due to the unknown rheology of the laboratory materials. We shall restrict the discussion below to experiments with Newtonian fluids, easier to control.

Whitehead and Luther (1975) performed experiments under normal gravity and showed that the morphology of the rising pockets of light fluid depends on the viscosity ratio between the two fluids: more viscous instabilities develop in finger-like ‘diapirs’, while less viscous instabilities develop into mushroom-shaped ‘cavity plume’ (Figure 18). The latter case is of particular interest for the initiation of instabilities deep in the mantle since hot mantle material is probably also less viscous (e.g., Stacey and Loper, 1983). The characteristic spacing \( \lambda \) and growth rate \( \sigma \) of RTI strongly depend on the viscosity of the materials (Selig, 1965; Whitehead and Luther, 1975; Canright and Morris, 1993; Bercovici
and Kelly, 1997; Ribe, 1998), as well as on the geometry of their source (Kerr and Lister, 1988; Lister and Kerr, 1989; Wilcock and Whitehead, 1991). For a thin denser lower layer of thickness $h_d$ and viscosity $\nu_d$ underlying an infinite layer of viscosity $\nu_m$, they scale as:

$$\lambda \sim h_d \ F(\gamma)$$

and

$$\sigma \approx \frac{\Delta \rho g h_d}{\rho \nu} \ G(\gamma)$$

where $g$ is the gravity acceleration, $\gamma = \nu_d / \nu_m$ and $\nu = \max(\nu_d, \nu_m)$. Equation [5b] shows that the instability growth rate is limited by the fluid with the larger viscosity. The functions $F$ and $G$ depend on the viscosity ratio and the geometry of the system. For a thin plane more viscous layer, linear stability analysis shows that $F(\gamma) \sim \gamma^{1/5}$ and $G(\gamma) \sim 1$, while for a thin plane less viscous layer, $F(\gamma) \sim \gamma^{-1/3}$ and $G(\gamma) \sim \gamma^{1/3}$ (Whitehead and Luther, 1975; Selig, 1965). Laboratory measurements agree with these scalings (Figure 19), which also confirms the ability of the linearized equations to predict the dominant wavelength of the developed instability (Whitehead and Luther, 1975). However, for instabilities developing from a cylinder in a denser fluid, the RTI spacing and growth rate still depend on the cylinder size but are independent of the viscosity ratio (Figure 19; Kerr and Lister, 1988; Lister and Kerr, 1989). For a thin layer of light fluid embedded in denser fluid, linear stability again predicts a fastest growing wavelength much greater than the thickness of the low-viscosity layer, which agrees with the experimental results (Figure 19; Lister and Kerr, 1989; Wilcock and Whitehead, 1991). Moreover, the displacement of the lower interface can reach a significant fraction of that of the upper interface. This leads to entrainment of the substratum into the light diapir (Ramberg, 1972; Wilcock and Whitehead, 1991). For very thin layers, a second instability is also observed at a scale much greater than the
characteristic wavelength; it arises from perturbations predominantly involving thickening rather than translation of the buoyant layer (Wilcock and Whitehead, 1991).

Bercovici and Kelly (1997) described both theoretically and experimentally the evolution of the interface from the initial linear instability to the formation and lift-off of cavity plumes. This involves nonlinear feedback mechanisms between the growth of the instability and the draining of the low-viscosity channel, whereby the proto-plume can stall for a long period of time before it separates and begins its ascent. The radius of the cavity plume $a$ and its trailing conduit $R$ now scale as

$$a \sim b_0 \gamma^{-2/9} \quad \text{and} \quad R \sim b_0$$

which shows that the radius of the head will be significantly larger than the conduit radius.

After RTI lift-off, the plumes may not always continue to rise along a vertical line as solitary bodies. Depending on their relative size, and the vertical and horizontal separation between them, they can become attracted to one another, clustering and even coalescing (Kelly and Bercovici, 1997). Combining theoretical, numerical, and laboratory results, Manga (1997) further showed that attraction between plumes was enhanced by plumes’ ability to deform (see Chapter 7.04).

Once the instabilities have started, the light fluid will gather at the top of the box, where it will remain since the density configuration is now stable. An initially unstable compositional stratification will therefore go unstable only once. In that respect, the Rayleigh–Taylor instability appears as an essential process of segregation of two intermingled materials. But the ‘one-shot’ character of RTI can be altered when the lighter fluid is continuously released from the bottom through a diffusive interface (Loper et al., 1988; Jellinek et al., 1999) or through melting (Jellinek et al., 1999). By introducing a silk membrane on the interface between a thin water layer and a heavy viscous corn syrup layer, Loper and McCartney (1986) and Loper et al. (1988) observed intermittent RTI generation. When the lighter fluid is less viscous, the RTI morphology is 3-D as already described, whereby it is sheetlike when the lighter fluid is more viscous (Jellinek et al., 1999). Moreover, for high release flux (or high Reynolds number), the RTI can entrain on its way up a significant amount of the ambient denser fluid, and the final stratification is significantly altered by mixing compared to the initial stage (Jellinek et al., 1999).

More commonly, episodicity can be sustained indefinitely when the density field is diffusing. This is the case in thermal convection, to which we turn now.

### 7.03.5 Simple Rayleigh–Benard Convection Studies

In buoyancy-driven flows, the exact governing equations are intractable. Some approximation is needed, and the simplest one which admits buoyancy is the Oberbeck–Boussinesq approximation (see Chapter 7.02). It assumes that

1. In the equations for the rate of change of momentum and mass, density variations may be neglected except when they are coupled to the gravitational acceleration in the buoyancy force. In practice, it requires $n \Delta T \ll 1$.
2. All other fluid properties can be considered as constant over the experimental cell.
3. Viscous dissipation is negligible.

![Figure 19](image.png)
In this section, we focus on cases where the Oberbeck–Boussinesq approximation is valid. Since mantle material has a high $Pr$, emphasis will be on results using high $Pr$ fluids. The convective flows have usually been determined using planforms visualized by shadowgraph, heat flow measurements, or time series of temperature measurements within the experimental cell.

7.03.5.1 Convection at Relatively Low $Ra(Ra_c \leq Ra \leq 10^6)$

As predicted by linear stability theory (Rayleigh, 1916; Chandrasekhar, 1961), convection should set in above a critical value $Ra_c$, which depends on the boundary conditions (see Chapter 7.04). Figure 20 shows a regime diagram of thermal convection as a function of Rayleigh and Prandtl numbers when both the top and bottom boundaries are rigid (zero velocity on the boundary) and isothermal. Rayleigh–Bénard convection exhibits a sequence of transitions toward chaos as $Ra$ increases. These transitions can also be seen on the measurements of the heat flux $Q_S$ extracted by the system (Figure 21), compared to the conductive heat flux $\frac{k \Delta T}{H}$. Their ratio defines the Nusselt number

$$Nu = \frac{Q_S}{\frac{k \Delta T}{H}}$$

It is equal to unity for conduction and exceeds unity as soon as convection starts. It was by measuring $Q_S$ as a function of $\Delta T$ that Schmidt and Silverton (1935) determined experimentally for the first time the critical Rayleigh number for the onset of convection. They confirmed the predicted theoretical value of 1708 with an accuracy better than 10%. Among others, Silveston (1958) extended the measurements to a wider range of fluids (Figure 21) and showed that, as predicted, the value of $Ra_c$ does not depend on $Pr$. However, this is not the case for the subsequent transitions between patterns at higher $Ra$ (Figure 20), for which there is a clear difference between low $Pr$ fluids (e.g., water, air, mercury, compressed gases like helium) and fluids with $Pr \geq 100$.

For $Pr \geq 60$, laboratory experiments performed with silicone oils revealed the following sequence of patterns in the convective planform as $Ra$ increases (Busse and Whitehead, 1971; Krishnamurti, 1970a, 1970b; Richter and Parsons, 1975): 2-D rolls (Figure 22(a)) are stable only for Rayleigh numbers less than 13 times critical. For $Ra$ greater than this, a second set of rolls perpendicular to the original ones grows, and the new planform is rectangular. In this so-called ‘bimodal’ regime (Figure 22(b)), the upwellings assume the geometry of two adjacent sides of a square, while the downwellings form the

![Figure 20](image-url) Regimes diagram of thermal convection. Data compiled from Krishnamurti (1970a, 1970b, 1979), Whitehead and Parsons (1978), Nataf et al. (1984), Guillou and Jaupart (1995), Zhang et al. (1997), Manga and Weeraratne (1999), and Xi et al. (2004). The square bars show the range of experiments performed. The experiments performed with a viscosity contrast within the tank greater than 10 are enclosed in the dotted line. The three round dots show experiments in centrifuge by Nataf et al. (1982).
other two sides of the square. At $Ra$ about 100 times critical, a new planform develops through a collective instability: the corners of the bimodal rectangles join, and the resulting planform is time dependent and resembles spokes radiating out of the central upwellings or downwellings (multimodal regime, Figure 22(c)). The two transitions in patterns are correlated with kinks in the $Nu-Ra$ curves (e.g., Malkus, 1954; Willis and Deardorff, 1967; Krishnamurti, 1970a, 1970b), and the $Ra$ value at each transition does not vary systematically with $Pr$ (Busse and Whitehead, 1971; Krishnamurti, 1970a, 1970b). However, the transition toward the time-dependent spoke pattern is not sharp, but there is a domain in $Ra$ where both time-dependent and stable cells can coexist. Hence, the pattern will depend strongly on the initial conditions. Under controlled initial conditions, Whitehead and Parsons (1978) observed stationary bimodal convection up to $Ra = 7.6 \times 10^5$, but an oscillating spoke pattern if starting from random conditions. At $Pr = 10^6$ (using a centrifuge), Nataf et al. (1984) observed steady motions at $Ra = 0.95 \times 10^5$, but time-dependent instabilities at $Ra = 6.3 \times 10^5$. When the Oberbeck–Boussinesq approximation applies, cold and hot currents have symmetric characteristics, even for time-dependent spokes. The horizontally averaged thermal structure is symmetric relative to the temperature at the mean depth (Figure 23).

For intermediate and low $Pr$, the pattern for a Boussinesq fluid always consists of straight rolls above but close to onset. However, the bifurcation toward stationary bimodal flow is replaced by an oscillatory secondary instability (e.g., Busse, 1978, 1989), and quite rapidly by chaos, then turbulence, as $Ra$ increases further (Figure 21).

**Figure 21** Nusselt number as a function of Rayleigh number measured in He. The different transitions observed are indicated. Adapted from Heslot F, Castaing B, and Libchaber A (1987). Transitions to turbulence in helium gas. Physical Review A 36: 5870–5873. The inset shows the data from Silveston (1958) around the convection onset.

**Figure 22** Shadowgraphs of the different stable cellular patterns in isoviscous convection at high $Pr$: (a) rolls, $Ra = 20 \times 10^3$; (b) bimodal, $Ra = 64 \times 10^3$ Instabilities of convection rolls in high Prandtl number fluid. Journal of Fluid Mechanics 47: 305–320; (c) spoke, $Ra = 1.4 \times 10^5$. (a) From Busse FH and Whitehead JA (1971) Instabilities of convection rolls in high Prandtl number fluid. Journal of Fluid Mechanics 47: 305–320 and (c) From Richter FM and Parsons B (1975). On the interaction of two scales of convection in the mantle. Journal of Geophysical Research 80: 2529–2541.
7.03.5.2 Patterns and Defects

The study of convection planforms and pattern selection is a very rich and fundamental field in itself, and has motivated a large body of theoretical (see Chapter 7.04) and experimental work. Recent reviews on the subject include papers by Busse (1978, 1989), Geitling (1998), Bodenschatz *et al.* (2000), and Ahlers (2005). Experiments are usually done in thin fluid layers with large aspect ratios (up to 100), and visualized by shadowgraph. We will mention here only a few striking results.

7.03.5.2.1 Generation of an initial prescribed pattern

To determine the domain of stability of the different patterns and follow the formation and evolution of the defects, the convective pattern must be initiated in a well-controlled way. The method developed by Chen and Whitehead (1968) consists in placing a grid made up of alternating blocked and clear areas over the top transparent channel. A 300–500 W incandescent lamp then is shone down through the pattern, so that the fluid lying below is slightly radiatively heated in the desired pattern. After a certain time (~1 h), the temperature of the top bath is decreased and the temperature of the bottom bath increased at equal rates (typically 2°min⁻¹) until the desired Rayleigh number is reached. It is important to change the baths at the same rates in order to exclude asymmetric or hexagonal modes of flow. Convection then starts with the location of the rising limbs of the cells being controlled by the places which were heated.

7.03.5.2.2 Patterns for Pr ≥ 100

Chen and Whitehead (1968) and Busse and Whitehead (1971) studied experimentally the stability of rolls for a silicone oil with Pr = 100. Rolls of known aspect ratio (roll width over tank thickness) were initiated, and allowed to evolve. The results were compared with the theory of Busse (1967a) which predicts a balloon-shaped region of stable rolls (Figure 24). The zigzag instability (Figure 25(a)) occurs when the aspect ratio is greater than 1 and tends to reduce it to 1. The cross-roll instability (Figure 25(b)) occurs when the aspect ratio is much less than 1, or when the Rayleigh number exceeds about $2 \times 10^9$: the initial rolls are replaced by two sets of rolls with different wavelength at right angles to each other. The pattern then evolves toward one set of rolls with increased aspect ratio (in the first case) or to the bimodal mode (in the second case). When two sets of rolls of
different wavelengths are combined, local rearrangement occurs through the pinching instability (Figure 25(c)). The agreement between the experimental results and the theory is quite remarkable (Figure 24; see also Chapter 7.04).

7.03.5.2.3 Patterns for Pr < 100

Patterns and defects have been extensively studied more recently, using primarily compressed gases as the working fluid (small cell sizes can be used and Pr can be tuned between 0.17 and 30), sensitive shadowgraph visualization coupled to digital image analysis, and quantitative heat flow measurements (for a review, see Bodenschatz et al. (2000)). Figure 25 shows examples of the patterns observed. Above but close to onset, the pattern for a Boussinesq fluid always consists of straight rolls, possibly with some defects induced by the sidewalls. At Pr ~ 1, the skewed-varicose instability arises to transform rolls into rolls of larger wavelength (Figure 25(d)). When non-Boussinesq conditions prevail, hexagons develop instead of rolls (Figure 25(e)). Further above onset, spiral-defect chaos (Morris et al., 1993) occurs in systems with Pr of order 1 or less (Figure 25(f)). This state is a bulk property and is not induced by side-walls. On the other hand, spirals occur in circular cells (Figure 25).

7.03.5.3 Thermal Convection at High Rayleigh Numbers (10^5 < Ra)

At high Ra, the convective pattern becomes disorganized and flow is generated locally by thermal boundary layer (TBL) instabilities (Elder, 1968), which develop into plumes (Figures 7 and 8). Their thermal anomalies are easily recorded by thermocouples located within the fluid (e.g., Townsend, 1957; Deardorff et al., 1969; Sparrow et al., 1970; Castaing et al., 1989; Davaille and Jaupart, 1993; Weeraratne and Manga, 1998; Lithgow-Bertelloni et al., 2001; Figure 28).

7.03.5.3.1 Thermal boundary layer instabilities

Plume generation in fully developed convection is a cyclic process in which the TBLs grow by thermal diffusion, become unstable when they reach a critical value.
thickness $\delta_c$ such that $Ra(\Delta T, \delta) = Ra_c$, and then empty themselves rapidly into plumes, at which point the cycle begins again (Figure 26). The characteristic timescale $\tau_c$ for this process is the time required for the growing TBL to become unstable, and is (Howard, 1964)

$$\tau_c = \frac{H^2}{\pi \kappa} \left( \frac{Ra_c}{Ra(H, \Delta T)} \right)^{2/3} \tag{8}$$

Because $Ra \sim H^3$ (eqn [1]), $\tau_c$ is independent of the layer depth $H$: the two TBLs do not interact anymore. This phenomenological model (Howard, 1964) and the scaling [8] is confirmed by laboratory experiments (Figures 26 and 27; e.g., Sparrow et al., 1970; Davaille and Jaupart, 1993; Davaille, 1999a; Manga et al., 2001; Davaille and Vatteville, 2005). Experiments further show that the typical spacing between plumes is 3–6 times $\delta_c$ (e.g., Sparrow et al., 1970; Tamai and Asaeda, 1984; Asaeda and Watanabe, 1989; Davaille et al., 2002; Jellinek and Manga, 2004). Moreover, the plumes seem connected by a network of ‘ridges’ near the base of the layer (Tamai and Asaeda, 1984; Asaeda and Watanabe, 1989). This morphology was confirmed by detailed numerical studies (e.g., Houseman, 1990; Christensen and Harder, 1991; Tackley, 1998; Trompert and Hansen, 1998a; Parmentier and Sotin, 2000). Plumes have a head-and-stem structure, and both head and stem diameters decrease with increasing $Ra$ (Lithgow-Bertelloni et al., 2001). Plumes can eventually detach from the TBL once they have drained it (Figure 26). They are transient features with a lifetime comparable to $\tau_c$ (Davaille and Vatteville, 2005). Since the two TBLs do not interact, the global heat flow should not depend on $H$ anymore, implying

$$Nu \sim Ra^{1/3} \tag{9}$$

This scaling was verified by Globe and Dropkin (1959) and Guillou and Jaupart (1995) using silicone oils for $Ra$ up to $Ra \sim 2 \times 10^7$. Goldstein et al., (1990), who employed an electrochemical analog to

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**Figure 26** Thermal boundary layer instabilities in a layer of sugar syrup, initially at 21°C and suddenly heated from below at 53°C ($Ra = 1.7 \times 10^6$). (a) Negative of the picture taken at $t = 300$ s. The isotherms appear as white lines. The TBL is growing by conduction from the lower boundary. (b) $t = 400$ s. The TBL becomes unstable. (c) $t = 460$ s. A thermal plume, outlined by the 24.6°C isotherm, rises from the TBL, and the TBL begins to shrink. (d) Corresponding velocity field deduced from PIV. The color background represents the velocity magnitude. (e) $t = 500$ s. The plume is well developed and the TBL is emptying itself in it. (f) Corresponding velocity field. (g) $t = 600$ s. The plume head has reached the upper boundary and begins to spread under it. The TBL has nearly disappeared and the conduit is disconnected from its source. (h) Corresponding velocity field. From Davaille A and Vatteville J (2005). On the transient nature of mantle plumes. Geophysical Research Letters 32, doi:10.1029/2005GL023029.
convection to reach $Ra \sim 10^{12}$ with $Pr = 2750$, recovered also (9). This regime of plumes has sometimes been named ‘soft turbulence’.

7.03.5.3.2 Soft and hard turbulence at low $Pr$

Steady-state experiments by Heslot et al. (1987) and Castaing et al. (1989) in helium ($0.6 \leq Pr \leq 1.7$) indicated that the spectrum of temperature fluctuations changes above $Ra = 4 \times 10^7$, defining a new dynamical regime called ‘hard turbulence’. This transition is also seen on the $Nu$–$Ra$ relationship (Figure 21), which follows the 1/3 power law [9] for $Ra = 4 \leq 10^7$ (the ‘soft turbulence’ regime), but changes to a 2/7 power law for $Ra \geq 4 \times 10^7$. On the other hand, Katsaros et al. (1977) recovered the 1/3 power law [9] for $Ra$ up to $10^9$ during the transient cooling from above of a water tank. Castaing et al. (1989) suggested that, in steady state, the dynamics of boundary layer instabilities were affected by the establishment of a large-scale circulation over the whole tank. Since the experiments were done in a container with aspect ratio 1, the large-scale circulation could have been caused by the wall effects. However, the establishment of a large-scale circulation had already been observed by Krishnamurti and Howard (1981) for larger aspect ratio. Since these experiments, much theoretical and experimental work has been done to confirm or disprove, extend, and explain the new regime (for recent reviews, see Sigigia (1994), Grossmann and Lohse, (2000); Chavanne et al. (2001)). It was found that a large-scale forced flow can decrease the convective heat flow (Solomon and Gollub, 1991). For aspect ratios smaller than 1, several regimes seem to coexist, due to the geometry of the large-scale circulation which can switch from one cell to several superimposed cells (e.g., Chilla et al., 2004). In the search for the ultimate turbulent regime, $Ra \sim 10^{17}$ has now been obtained using cryogenic He (Niemela et al., 2000). However, from $10^6$ to $10^{17}$, these authors found neither a 2/7 nor a 1/3 power law, but a 0.30 power-law. Following Kraichman (1962), Grossmann and Lohse (2000) proposed a systematic theory to predict all the different asymptotic regimes as a function of $Ra$ and $Pr$. To be able to compare accurately theory and experiments, special care is now taken to estimate heat losses from the sides (e.g., Alhers, 2001) and from the finite conductivity of the bottom and top plates (e.g., Brown et al., 2005; see Section 7.03.2.3), as well as the $Pr$ variations. However, it seems that the ‘ultimate’ regime has not been found yet.

7.03.5.3.3 Large-scale (cellular) circulation and plumes at high $Pr$

To obtain high Rayleigh number in high $Pr$ fluids requires large temperature differences (typically $\geq 20^\circ$C) and large depths (typically $\geq 20$–30 cm). The aspect ratios of these experiments are therefore small (between 1 and 3 at most). Even for fluids with very weak temperature dependence of viscosity (e.g., silicone oils), the viscosity can no longer be treated as constant (Figures 2 and 20). Its value is therefore taken at the mean of the boundary temperatures, $(T_{bot}+T_{top})/2$.

Weeraratne and Manga (1998), Manga and Weeraratne (1999), and Manga et al., (2001) determined the different styles of time-dependent convection from temperatures recorded on the tank mid-plane (Figure 28). For steady flow ($Ra \leq 10^9$), these remain constant. When the flow first becomes unsteady, temperature fluctuations with large amplitude and long period appear (Figure 28(a)). As $Ra$ increases, the amplitude of long-period fluctuations decreases and small short-period fluctuations appear (e.g., $Ra \sim 5 \times 10^6$, Figure 28(b)). At still higher $Ra$, the large-amplitude fluctuations disappear completely and only the short-period fluctuations remain (Figure 28(c)). As mushroom-shaped plumes were

![Figure 27](image-url)
observed in this regime, the small-scale fluctuations were interpreted as their signature, and the long-period signal as the signature of large-scale (cellular) flow. From $Ra \sim 10^7$ to $2 \times 10^8$, no large-scale circulation was observed. Such circulation was also absent from transient experiments where the fluid was continuously heated from below (Lithgow-Bertelloni et al., 2001) or cooled from above (Davaille and Jaupart, 1993), and the other boundaries were insulated. Using glycerol (Zhang et al., 1997, 1998) and dipropylene glycol (Xi et al., 2004) in aspect ratio 1 cells, a different picture was found for $Ra \sim 10^8-10^9$: a large-scale circulation develops which entrains the plumes as soon as they have formed. Measuring the velocity field by PIV, Xi et al. (2004) demonstrated that the flow becomes organized because of the plumes, which interact through their velocity boundary layers typically as soon as they have risen a distance comparable to their spacing: plume clustering then occurs, as in Rayleigh–Taylor instabilities (Bercovici and Kelly, 1997) and in experiments on the interaction between two thermal plumes (Moses et al., 1993). In this regime where plumes and large-scale circulation coexist, the exponent of the $Nu-Ra$ power law becomes again closer to $2/7$ (Xi et al., 2004). It would now be interesting to run the same experiments with larger aspect ratio to determine if the large-scale circulation always spans the whole tank, or if it has its own wavelength.

### 7.03.5.4 Convective Regime in an Isoviscous Mantle

From Figure 20 and eqn [1], one can determine the convective regime of a mantle layer as a function of its thickness and average viscosity (Figure 29). Plumes will develop in the upper mantle (of thickness 660 km) only if its viscosity is lower than $10^{21}$ Pa s. For an average mantle viscosity of $10^{22}$ Pa s, plumes will be generated only if the mantle layer thickness exceeds 2000 km. Convective motions originating at the CMB and developing over the whole mantle thickness should therefore take the form of plumes (Parmentier et al., 1975; Loper and Stacey, 1983). However, those plumes are transient phenomena. According to eqn [8], $\tau_c \sim 10$ My, 40 My and 200 My for dynamic viscosities $\nu_m = 10^{19}$ Pa s, $5 \times 10^{20}$ Pa s, and $10^{22}$ Pa s, respectively. Such recurrence times are probably also upper bounds on plume lifetime. The corresponding values of the critical TBL thickness $\delta_c \sim H(Ra_c/Ra)^{1/3}$ for the same viscosities are 31, 62, and 140 km, respectively, which

![Figure 28](image-url)  

**Figure 28** Temperature fluctuations $\theta_m = (T - T_{top})/\Delta T$ at three thermocouples located at mid-height in a tank of convecting corn syrup. Time is normalized by the diffusive timescale $\theta^2/\kappa$. (a) Unsteady convection dominated by large-scale flow ($Ra = 2.5 \times 10^5$), (b) Increased plume activity along with large-scale flow ($Ra = 4.7 \times 10^6$), and (c) Plume dominated convection with short period fluctuations ($Ra = 5.5 \times 10^7$). From Weeraratne D and Manga M (1998) Transitions in the style of mantle convection at high Rayleigh numbers. *Earth and Planetary Science Letters* 160: 563–568.

![Figure 29](image-url)  

**Figure 29** Convective regime developing in a mantle layer as a function of its thickness and viscosity. The pattern is cellular for $Ra \gtrsim Ra_c = 650$, and in plumes for $Ra \gtrsim 10^6$. In gray the average viscosity inferred for the whole mantle (Kohlstedt, volume 2). The thin gray line delimits the upper mantle. The calculation has been done from eqn [1] taking $\kappa = 10^{-6}$ m$^2$ s$^{-1}$, $\alpha = 2 \times 10^{-5}$ K$^{-1}$ and $\Delta T = 3000$ K.
implies a plume spacing 100–840 km in the mantle. Given the 2900 km thickness of the mantle, plume clustering could occur (e.g., Schubert et al., 2004). Moreover, from Figure 26, we expect that a plume will eventually detach from the hot TBL from which it originated. This implies that the absence of a plume conduit in a tomographic image need not indicate the absence of a plume.

However, if isoviscous convection predicts plumes, it does not predict plates, which are the main signature of convection on the Earth’s surface (e.g., Chapters 7.01 and 7.02). Part of the difficulty in determining the convective pattern in the mantle resides in the complexity of mantle rheology, which depends on temperature (strongly), pressure, partial melting (strongly), water content (strongly), chemistry and strain rate (see Chapter 2.14). The rheology of the lithosphere is especially difficult to quantify since it comprises a brittle skin as well as a viscous lower part. The convective pattern driven by the ‘cold’ upper-mantle boundary appears to be cellular, although time dependent. Inverting Figure 29 for this regime, it would therefore indicate an ‘effective’ mantle viscosity greater than $10^{23}$ Pa s, that is, much lower than the viscosity of the cold lithosphere, but at least an order of magnitude higher than the measured bulk mantle viscosity. This asymmetry in mantle convection could be caused by its mixed heating mode (internal and bottom heating), and by its rheology, in particular the temperature dependence of viscosity. We now turn to the experiments which have been devoted to these two effects.

### 7.03.6 Temperature-Dependent Viscosity

A strong temperature dependence of the viscosity breaks the symmetry in the convection cell: cold instabilities are now more viscous than the fluid in the bulk of the tank, while the hot instabilities are less viscous. According to what has been observed in RTI (Section 7.03.4, Figure 18), their morphology should change. Figure 30 shows that this is indeed the case: hot TBL instabilities are mushroom shaped, while cold instabilities become diapirs, which can fold on the bottom plate before spreading (Figure 30(b)). Material in the cold TBL also flows less easily and rapidly than in the rest of the fluid, which will diminish the efficiency of convection and the heat transport. The system is now characterized by $Ra$ (based on the viscosity at the mean of the boundary temperatures), $Pr$, and the viscosity ratio $\gamma = \eta(T_{top})/\eta(T_{bot})$.

#### 7.03.6.1 Rigid Boundaries

**7.03.6.1.1 Onset of convection**

Stengel et al. (1982) studied theoretically and experimentally the onset of convection for $\gamma \leq 3 \times 10^3$. $Ra_c$ remains constant at low $\gamma$, increases at moderate $\gamma$ to a maximum for $\gamma \approx 3000$, and then decreases. According to Stengel et al. (1982), this occurs because convection begins first in a (hotter) less viscous sublayer where the local Rayleigh number is maximum. Richter et al. (1983) increased the range of viscosity ratio to $10^5$, and found similar results. Moreover, finite-amplitude disturbances are seen to grow at Rayleigh numbers below critical, as predicted by Busse (1967b) for fluids with weakly temperature-dependent viscosity.

**7.03.6.1.2 Patterns and regimes**

Richter (1978) explored the stability of rolls, using glycerine and L100 (polybutene), for $Ra \leq 25 \times 10^3$ and $\gamma \leq 20$. For large $\gamma$, hexagons were found to be the stable pattern. White (1988) used Golden Syrup...
to study the stability of patterns for $\gamma \leq 1000$ and $Ra \leq 63 \times 10^3$ (Figure 31). Besides rolls and hexagons, a new planform of squares (Figure 31(b)) was found to be stable at large viscosity variations, and pattern instabilities developed in a mosaic form (Figure 31(e) and 31(f)). At fixed wave number, rolls are always unstable as soon as $\gamma > 40$. Moreover, squares break down to a spoke pattern around $Ra = 20,000$, that is much sooner than in the constant viscosity case (transition at $Ra \sim 10^3$). The wavelength of squares is also observed to decrease with increasing $\gamma$. Temperature profiles through the

Figure 31 Convective patterns in fluids with strongly temperature-dependent viscosity (Golden Syrup). (a) Stability map for rolls, hexagons, and squares. There is a large amount of hysteresis where the realized planform depends on what pattern was originally induced. (b) Squares. (c) Hexagons. (d) Triangles. (e, f) Mosaic instabilities. From White DB (1988) The planforms and onset of convection with a temperature-dependent viscosity fluid? Journal of Fluid Mechanics 191: 247–286.
layer (e.g., Figure 22) revealed that this shift is associated with the development of a thick, cold boundary layer, and an increase of the interior temperature (White, 1988; Richter et al., 1983). The temperature profiles allowed the calculation of the convective heat flow and viscosity profile, which showed that most of the temperature variations across the tank occur in the cold TBL, and that the top part of the latter becomes stagnant (zero convective heat flow). Hence, convection develops in a sublayer where the viscosity ratio reaches at most 100 (Richter et al., 1983). This stagnant-lid regime has been observed for $Ra$ up to $10^5$ (Davaille and Jaupart, 1993).

Global heat flow measurements were carried out by Booker (1976), Booker and Stengel (1978), and Richter et al. (1983). For the entire range of parameters ($10^5 \leq Ra \leq 6 \times 10^8$ and $\gamma < 10^3$), the heat transfer differs little from that of a uniform-viscosity fluid when the Rayleigh number is defined with the viscosity at the mean of the boundary temperatures. The relationship between Nusselt number and supercriticality ($Ra/Ra_c(\gamma)$) is even more remarkable since it is independent of $\gamma$ and indistinguishable from the results of Rossby (1969) for constant viscosity:

$$Nu = 1.46 \left( \frac{Ra}{Ra_c(\gamma)} \right)^{0.281} \quad [10]$$

Subsequent high-quality heat flow measurements by Giannandrea and Christensen (1993) agree with this law. Weeraratne and Manga (1998), Manga and Weraratan (1999), and Manga et al. (2001) extended the range of $Ra$ up to $2 \times 10^8$ for $\gamma = 1800$. We already described in the previous section the sequence of regimes which they observed for $\gamma = 170$, that is, in absence of a conductive lid (Figure 20). Their heat flux data agree with eqn [10] for $Ra$ up to $2 \times 10^8$ (Figure 32). For higher $Ra$ and higher $\gamma$, the heat flux is less than that predicted by [10] and by numerical calculations at infinite $Pr$. This trend was attributed to inertial effects (Manga and Weraratan, 1999).

### 7.03.6.1.3 Characteristics of the stagnant lid regime

Davaille and Jaupart (1993, 1994) studied transient high $Ra$ convection in a layer of Golden Syrup suddenly cooled from above. With this setup, the viscosity contrast across the cold TBL, $\gamma_{TBL}$, ranged between 2 and $10^3$. For $\gamma > 100$, convection developed only in the lower part of the TBL, and the upper part remained stagnant (Figure 33). The Rayleigh number and all the scaling laws therefore used the viscosity of the well-mixed interior, since it would be meaningless to characterize the flow with a viscosity corresponding to a temperature well within the stagnant fluid. At the onset of convection, the viscosity contrast across the unstable region is about 3; in fully developed convection, it reaches a typical value of 10. The temperature difference $\Delta T_{eff}$ across the unstable part of the lithosphere depends only on the interior temperature $T_m$ and on a temperature scale controlled by the rheology $\Delta T_c$ (e.g., Morris, 1982; Morris and Canright, 1984; Davaille and Jaupart, 1993):

$$\Delta T_c(T_m) = \frac{\eta(T_m)}{[\partial q(\partial T)](T_m)}$$

Laboratory results then give $\Delta T_{eff} = 2.24 \Delta T_c$. A necessary condition for the existence of a stagnant lid is that the applied temperature difference exceeds a threshold value equal to $\Delta T_{eff}$. All the dynamics of the system are then determined locally in the unstable part of the TBL. Following once again the phenomenological model of Howard (1964), the heat flux can therefore be written as

$$Q_c = -Ck(\alpha g/\kappa T_m)^{1/3} \Delta T_c^{4/3} \quad [12]$$

where the constant $C = 0.47 \pm 0.03$ is determined experimentally. This model also predicts well the convective onset time and the amplitude of the temperature fluctuations (Davaille and Jaupart, 1993).
Subsequent numerical and experimental work also recovered the functional forms of [11] and [12], with $\Delta T_{\text{eff}} = 2.2 - 2.6 \times T_c$ and $C = 0.4 - 0.9$ (e.g., Grasset and Parmentier, 1998; Trompert and Hansen, 1998a; Dumoulin et al., 1999; Solomatov and Moresi, 2000; Manga et al., 2001; Gonnermann et al., 2004). In the case of steady Rayleigh–Bénard convection at high Ra, the temperature difference in the hot TBL has been found to scale as $(1.1 - 1.3) \times T_c$ (Solomatov and Moresi, 2000; Manga et al., 2001), which together with [11] allows one to predict the temperature of the well-mixed interior (Solomatov, 1995; Manga et al., 2001). Moreover, since the cold and hot unstable thermal boundary layers have different characteristics, plumes are released from these layers with different frequencies, and cold more viscous downwellings carry a greater temperature anomaly than their hot counterparts. Measurements show that whereas there is a single frequency for cold plume formation, hot plumes form with multiple frequencies and in particular may be triggered by cold sinking plumes (Schaeffer and Manga, 2001).

7.03.6.2 Plate Tectonics in the Laboratory?

Mantle convection is characterized by mobile plates on its top surface, which have no chance to be recovered in experiments with a rigid top boundary since the latter imposes a zero-fluid velocity on the surface. Moreover, 2-D numerical simulations had shown that the heat transport strongly depends on the mobility of the surface boundary layer (Christensen, 1985), which is controlled by the viscosity contrast. Giannandrea and Christensen (1993) and Weinstein and Christensen (1991) therefore carried out experiments in syrups in a large aspect ratio tank (Figure 39) to study the effect of a stress-free boundary (Figure 4(b)). They observed two regimes. For $\gamma \leq 1000$, the surface layer is mobile. At $Ra = 10^7$, the morphology of downwellings changes, from the spokes obtained with a rigid boundary, to a dendritic network of descending sheets, with a wavelength more than 3 times that of the spoke pattern (Figure 34). The existence of this ‘whole-layer’ or ‘sluggish lid’ mode with an increased wavelength is in agreement with earlier predictions (Stengel et al., 1982; Jaupart and Parsons, 1985) and 3-D numerical calculations (Ogawa et al., 1991). The Nusselt number drops by 20% for viscosity contrasts between 50 and 5000. For $\gamma \geq 1000$, a stagnant lid forms on the top of an actively convecting region; and both the Nusselt number and the size of the convection cells are nearly identical for both rigid and free boundaries.

Figure 35 shows the regime diagram established from experimental, numerical, and theoretical analysis (Solomatov, 1995). With a viscosity contrast across the lithosphere $\sim 10^7$, the Earth should be in the stagnant-lid regime, that is, a one-plate planet like Mars. So, if temperature-dependent viscosity is clearly a key ingredient for plate formation, this ingredient alone is not sufficient to generate ‘plate tectonics’ convection. To make plate tectonics work, a failure mechanism is needed and several options have recently been proposed (see Bercovici et al., 2000; Bercovici, 2003; see Chapters 7.02 and 7.05). The simplest is probably pseudo-plastic yielding (Moresi and Solomatov, 1998; Trompert and Hansen, 1998b; Tackley, 2000; Stein et al., 2004; Grigné et al., 2005): moving plates and thin...
weak boundaries appear but subduction remain symmetric. Gurnis et al., (2000) also pointed out the importance of lithosphere ‘memory’: the lithosphere can support dormant weak zones (faults or rifts) over long time periods, but those weak zones can be preferentially reactivated to become new plate boundaries. Another ingredient could be damage (e.g., Bercovici et al., 2001), which introduces some memory in the rheology and allows development over time of weak zones. This approach has been used with some success (e.g., Ogawa, 2003) but a physical understanding of the damage process from the grain to the lithosphere scale is still lacking. Anyway, experimentalists are still looking for a laboratory fluid presenting the ‘right’ kind of rheology to allow plate tectonics. However, the study of the stagnant-lid regime is still relevant for the dynamics of plate cooling, to which we turn now.

7.03.6.3 Stagnant-Lid Regime and Lithosphere Cooling

The oceanic lithosphere thickens at it moves away from a mid-ocean ridge (see Chapters 7.07 and 6.05). It has been proposed that cold thermal instabilities develop in its lower part. Such ‘small-scale convection’ (SSC) was first invoked to explain the flattening of heat flux and bathymetry of the oceans at old ages (Parsons and Sclater, 1977; Parsons and McKenzie, 1978) and later several other phenomena occurring on different time and length scales, such as small-scale (150–500 km) geoid anomalies in the central Pacific (e.g., Haxby and Weissel, 1986) and central Indian (Cazenave et al., 1987) oceans, differences in subsidence rates (Fleitout and Yuen, 1984; Buck, 1987; Eberle and Forsyth, 1995), ridge segmentation (e.g., Sparks and Parmentier, 1993; Rouzo et al., 1995), delamination of the lithosphere under hot spots (Sleep, 1994; Moore et al., 1998; Dubuffet et al., 2000), and patterns of anisotropy beneath the Pacific ocean (Nishimura and Forsyth, 1998; Montagner and Tanimoto, 1990; Davaille and Jaupart, 1994; Ekström and Dziewonski, 1998).
Davaille and Jaupart (1994) applied the scaling laws derived from their experimental results, using for the mantle a Newtonian creep law with activation enthalpy $H$:

$$\nu(T) = \mu_r \exp\left(\frac{H}{RT}\right)$$  \[13\]

For $H$ varying between 150 and 500 kJ mol$^{-1}$ (the value depends on the creep mechanism and the water content; see Chapter 2.14), $\Delta T_{\text{eff}}$ ranges between 310°C and 90°C according to [11] and [13]. Since small-scale instabilities develop when the local Rayleigh number based on the characteristics of the unstable part of the lithosphere (temperature, viscosity, thickness) exceeds a critical value, the onset of convection depends only on the thermal structure of the lithosphere, and on the rheology of mantle material. Following Davaille and Jaupart (1994), the onset time can be written as

$$\tau_c = \frac{a}{\kappa} \left(\frac{\alpha g \Delta T_{\text{eff}}}{\kappa \nu_m}\right)^{-2/3} \times \left[ 1 + \frac{b}{f(\Delta T/\Delta T_{\text{eff}})} \left(\frac{\Delta T}{\Delta T_{\text{eff}}} - 1\right) \right]^2$$  \[14\]

where $\nu_m$ is the viscosity of the asthenosphere at the mantle temperature $T_m$, $a = 51.84$ and $b = 0.3013$ are two constants determined from the laboratory experiments, and $\Delta T$ is the temperature drop across the lithosphere. The function $f$ depends on the cooling model and for a half-space conductive cooling is given in Figure 36(a). For a thin stagnant lid (or $\Delta T/\Delta T_{\text{eff}} < 3$), $f \sim 1$. This approximation applies well to laboratory experiments and was therefore adopted by Davaille and Jaupart (1994); thin line labeled DJ94 on Figure 36(b). However, as pointed out by subsequent studies (Dumoulin et al., 2001; Zaranek and Parmentier, 2004), $\Delta T/\Delta T_{\text{eff}} > 4$ for the mantle lithosphere and the approximation $f \sim 1$ leads to overestimate the SSC onset time (Figure 36(b)) compared to the prediction of [14] without the approximation (thick black line on Figure 36(b)). In the last years, several numerical studies, using either Arrhenius or exponential viscosity laws, have allowed to extend the range of $\Delta T/\Delta T_{\text{eff}}$ studied (Choblet and Sotin, 2000; Dumoulin et al., 2001; Korenaga and Jordan, 2003; Huang et al., 2003; Zaranek and Parmentier, 2004), and to develop new scaling laws. The discrepancies between the different data sets reveal the relative importance of the onset time measurement, the type of viscosity law (10–20% of change only), and the type of perturbation (initial-time/at-all-times, numerical-noise/finite-size-perturbation) introduced numerically which will grow toward convective instabilities. The latter has the strongest influence (Zaranek and Parmentier, 2004): the longest onset times are obtained when the numerical perturbations initially introduced are smallest (Choblet and Sotin, 2000). The trend of the laboratory experiments seems to compare best with numerical experiments which introduce thermal noise around $10^{-3} \times \Delta T$ at all times (Figure 36(b)). Then, in the parameter range relevant for the Earth’s mantle

![Figure 36](image-url)
(ΔT/ΔT_{eff} ≥ 4), numerical studies agree with [14] within 25%. So, if the magnitude of the temperature perturbations in the lithospheric mantle is similar to the laboratory one, an SSC onset time between 10 and 100 My is predicted, depending on the asthenospheric mantle temperature.

7.03.7 Complications: Internal Heating, Continents and Moving Plates

7.03.7.1 Internal Heating

We have so far focused on the dynamics of fluid layers uniformly heated from below and cooled from above. However, the Earth's mantle is not only heated along the core–mantle boundary by the hotter molten iron core, but is also partly heated from within by radiogenic decay of uranium, thorium, and potassium distributed throughout the mantle (see Chapter 7.06). Thermal convection with both internal and bottom heating is characterized by two dimensionless numbers: Ra for the bottom heating given by eqn [1] and an equivalent Rayleigh number for the purely internally heated case, defined as

\[ Ra_{H} = \frac{\alpha g H^{3} Q_{H}}{\nu k H} \]  

where Q_{H} is the volumetric rate of internal heat generation.

7.03.7.1.1 Purely internally heated fluid

The heat sources are simulated by the Ohmic dissipation of an electric current passed through an electrolyte fluid layer. Electrodes are placed either on each of the horizontal boundaries (e.g., Carrigan, 1982), or on two of the side walls of the chamber (e.g., Kulacki and Goldstein, 1972). Polarization effects in the fluid are eliminated by applying an alternating current. In all the experiments using this technique, the bottom of the tank is insulated (zero heat flux) and the top of the tank is cooled by a thermostated bath. One therefore expect only one TBL in the system, just below the cold plate.

Most of the experiments that have been performed used aqueous solutions. Tritton and Zarraga (1967) and Schwiderski and Schab (1971) visualized the planform of convection from marginal stability up to the transition to turbulence. Quasi-steady planforms were observed with downflow in the centers of the cells for Ra_{H} ≤ 80Ra_{c}. For larger Ra_{H}, cells broke up and turbulent motions appeared. Kulacki and Goldstein (1972) extended the range up to 675Ra_{c}, well into the turbulent regime. Kulacki and Nagle (1975) focused on heat flow measurements as a function of Ra_{H}. For the turbulent regime (1.5 × 10^{5} ≤ RaH ≤ 2.5 × 10^{9}), their data are consistent with a power law Nu ~ 0.305 Ra^{0.239}. As for the bottom heated case, transitions in the Nu = f(Ra_{H}) curves were observed at each change of the convective pattern. Thermal fluctuations were also found to be maximum just outside the upper cold TBL and no bottom hot TBL was observed.

Carrigan (1982) used glycerine (Pr ~ 3000) for 10^{3} ≤ Ra_{H} ≤ 10^{5}. He observed cold spoke patterns up to Ra_{H} = 108Ra_{c}, and a superposition of a large-scale circulation and cold individual plumes for larger Ra.

7.03.7.1.2 Bottom- and internally-heated fluid

Following Krishnamurti (1968a), Weinstein and Olson (1990) demonstrated the equivalence between thermal convection in a plane layer of fluid with uniformly distributed heat sources between boundaries with constant temperatures, and thermal convection in a plane layer of fluid with boundary temperatures that are spatially uniform but vary in time at a rate dT_{top}/dt. The effective volumetric internal heat generation is then

\[ Q_{H} = -\rho c_{p} \frac{dT_{top}}{dt} \]  

This technique is simpler and more flexible to use than the electrolytic one, since it can be used with any fluid and allows the fluid to be heated from below as well as internally. Krishnamurti (1968b) performed experiments with silicone oils (Pr ~ 200) near the onset of convection. Using high Prandtl number oils (Pr ~ 9 × 10^{3}), Weinstein and Olson (1990) studied the evolution of the convective pattern and its time dependence as the proportion of internal heating increases (Figure 37). At Ra = 1.5 × 10^{5}, in the absence of internal heat generation, a spoke pattern is observed, in which connected spokes of ascending and descending flows are on average of equal strength. With internal heat generation, the buoyancy becomes concentrated into the cold downwellings, and the amount of time dependence increases. Hot active instabilities will reappear if there exists a bump on the hot bottom boundary with a height comparable to (or greater than) the TBL thickness (Namiki and Kurita, 2001). In this case, hot plumes will be anchored on the bump. So although an internally heated mantle favors
cold downwellings, hot plumes can still develop, provided that the bottom of the mantle, or the $D^0$ layer, has some topography.

7.03.7.2 Inhomogeneous Cooling/Continents

Continental lithosphere is thicker than its oceanic counterpart, and does not subduct. Its crust is also richer in radiocative elements. Moreover, heat flow beneath continents is significantly lower than under oceans (e.g., Jaupart et al., 1998), which tends to show that continents are more insulating. Hence, continents induce a lateral heterogeneity in both the mechanical and thermal conditions at the upper boundary of the mantle. The experimental study of Rossby (1965) showed that any lateral heterogeneity in cooling or heating will generate an horizontal large-scale flow. Elder (1967) was probably the first to suggest that the continents, by acting as an insulating lid, could generate a new class of lateral motions. Later studies have therefore focused on characterizing and quantifying the influence of these lateral heterogeneities on mantle thermal convection.

Whitehead (1972, 1976) investigated both theoretically and experimentally the conditions under which a heater mounted on a mobile float could become unstable to drifting motion. For cellular convection, the drift was found of the same magnitude as the characteristic convective velocity.

Guillou and Jaupart (1995) investigated the thermal effect of a fixed continent on the convective pattern and heat flow, using silicone oils and Golden Syrup for $Ra$ ranging between critical and $10^6$. The continent was modeled as an insulating material embedded into a cold copper plate (Figure 38), and its size and shape were varied. They observed the generation of deep hot instabilities localized under the continent and the generation of cold instabilities outside or on its edges (Figure 38). The presence of a continent therefore shields part of the underlying mantle from subduction (Gurnis, 1988), and also induces a large-scale convection pattern oceanward over distances up to 3 times the continent’s size (Guillou and Jaupart, 1995). Combining experiments, theory, and simulations, Lenardic et al. (2005) studied the influence of partial insulation on mantle heat extraction. Since partial insulation leads to increased internal mantle temperature and decreased viscosity, it allows, in turn, for the more rapid overturn of oceanic lithosphere and increased oceanic heat flux. For ratios of continental to oceanic surface area lower than 0.4, global mantle heat flow could therefore remain constant or even increase as a result. Wenzel et al. (2004) used the same kind of setup with two-layer convection. They suggested that, on Mars, the combination of crustal dichotomy (the martian crust is much thicker, therefore insulating, in the southern hemisphere than in the northern hemisphere) and two-layer convection would lead to the formation of a large-scale upwelling under the southern highlands that appeared early and endured for billions of years.

Zhang and Libchaber (2000) and Zhong and Zhang (2005) replaced the rigid cold boundary by a free surface cooled by air, which allowed the insulating raft to move freely in response to the convection (Figure 39). The experimental fluid was water, with Rayleigh numbers ranging from $10^7$ to $4 \times 10^8$. Like Guillou and Jaupart (1995), they observed that the insulating raft distorts the local heat flux and induces a coherent flow over the experimental cell, which in turn moves the plate. In their confined geometry, the plate can be driven to a periodic motion even under the action of chaotic convection. The period of

**Figure 37** Shadowgraphs of the planform of thermal convection with internal heat generation for $Ra = 1.5 \times 10^5$ (spoke pattern). (a) $Ra_H/Ra = 0$; light and dark lines have the same intensity as cold and hot currents have the same strength. (b) $Ra_H/Ra = 6$; the dark lines are fading as the downwellings become weaker. (c) $Ra_H/Ra = 18$; no hot upwellings are seen anymore. From Weinstein SA and Olson P (1990) Planforms in thermal convection with internal heat sources at large Rayleigh and Prandtl numbers. Geophysical Research Letters 17: 239-242.
oscillation depends on the coverage ratio and on the Rayleigh number (Figure 39).

Nataf et al. (1981) investigated the influence of the sinking cold oceanic lithosphere on the neighboring subcontinental convection and on the generation of instabilities in the horizontal boundary layers by performing experiments with cooling from both the top plate and one of the sides. The system dynamics are then characterized by two Rayleigh numbers: $Ra$ based on the vertical temperature difference, and a lateral Rayleigh number based on the lateral temperature difference ($Ra_{lat}$). The lateral cooling induces a large roll with axis parallel to the cold wall and descending flow next to it. At constant $Ra$, the width of the large roll is proportional to $Ra_{lat}^{1/2}$, while for constant $Ra_{lat}/Ra$, it scales as $\sim Ra^{1/6}$. In the upper mantle such large-scale circulation could become 5 times wider than deep. The experiments also show that hot instabilities cannot develop directly beneath the cold downwelling flow, because the spreading of the latter impedes the growth of the hot TBL.

7.03.7.3 Moving Top Boundary

We just saw that any large-scale circulation strongly influences the generation of convective features of smaller extent. The large-scale motion of plates is therefore expected to influence all motions within the mantle, especially small-scale convection under the cooling lithosphere, and the distribution of hot spots.
Following the theoretical study of Richter (1973), Richter and Parsons (1975) studied the influence of a moving top boundary on the convective pattern for $Ra$ between $3.3 \times 10^5$ (rolls) and $1.4 \times 10^7$ (spokes). A mylar sheet was introduced just below the top cold boundary and pulled at constant velocity $U$ (Figure 4). Besides $Ra$, another system now controls the system dynamics, the Péclet number $Pe = UH/\kappa$, which compares the heat transport by advection and by diffusion. It ranges between 30 and 720 in the experiments. Initially, the top boundary was at rest until the convective pattern becomes quasi-steady. For the lower $Ra$, rolls orthogonal to the spreading direction were initiated using the technique of Chen and Whitehead (1968). Then the velocity of the mylar was set to a finite value and the convective pattern followed by shadowgraph. Over the whole $Ra$ range, the pattern was observed to evolve toward rolls parallel to the mylar velocity. Scaling laws show that it would probably take 70 My for the geometry of convection to reorganize into rolls parallel to the spreading direction over the whole thickness of the upper mantle.

Experiments of Kincaid et al. (1996), Jellinek et al. (2002, 2003), and Gonnermann et al. (2004) studied how a plume-dominated free convective flow interacts with a large-scale passive flow driven by plate motions. Kincaid et al. (1996) focused on the interaction of mid-ocean ridges and convection in the upper mantle. The three other papers extended this study to the whole mantle, higher $Ra$, and fluids with strongly temperature dependent viscosity. Jellinek et al. (2002, 2003) performed laboratory experiments using corn syrup heated from below and driven at its surface by a ridge-like velocity boundary condition (Figure 40(a)). They documented three different flow regimes, depending on the ratio $\nu$ of the spreading rate to the typical plume rise velocity and the ratio of the interior viscosity to the viscosity of the hottest fluid $\gamma$. When $\nu < 1$ and $\gamma \geq 1$, the plume distribution is random and unaffected by the large-scale flow. The opposite extreme is represented by $\nu > 10$ and $\gamma > 100$, where plume formation is suppressed entirely and the large-scale flow carries all the heat flux. Interaction of the two flows occurs for intermediate values of $\nu$; established plume channels are now advected along the bottom boundary and focused toward an upwelling beneath the ridge. The basal heat flux and the TBL thickness depend on $Pe$ and $\gamma$. In particular, since cold, and therefore also more viscous, fluid is rapidly transported from the top to the bottom of the layer, the viscosity contrast across the lower TBL increases, generating hot plumes with a bigger head to tail ratio than without plate circulation (Nataf, 1991; Jellinek et al., 2002). Running more detailed experiments, Gonnermann et al. (2004) proposed a conceptual framework (Figure 40(b)) whereby the tank bottom can be divided in four regions. The large-scale flow impedes the growth of the hot TBL where the cold and more viscous downwelling spreads (Nataf et al., 1981, and therefore the hot TBL is not thick enough in region I to develop plumes in III, while TBL instabilities develop in regions II and IV. From Gonnermann HM, Jellinek AM, Richards MA, and Manga M (2004) Modulation of mantle plumes and heat flow at the core mantle boundary by plate-scale flow. Results from laboratory experiments. Earth and Planetary Science Letters 226: 53–67.
7.03.8 Close-Up on Plumes: Plumes from a Point Source of Buoyancy

Since the mid-1970s, much of the best experimental work in mantle dynamics has been devoted to plumes from point sources of buoyancy. While plumes in the mantle probably develop primarily as instabilities of thermal boundary layers (e.g., Parmentier et al., 1975; Loper and Stacey, 1983), such instabilities are time dependent and can be difficult to quantify (see Section 7.03.5). As a first step, therefore, it makes sense to investigate the simpler case of an isolated laminar ‘starting plume’ rising from a point source of buoyancy whose strength is constant in time. Such plumes are easily studied and photographed in the laboratory, and have given us some of the most beautiful images in the field of experimental geodynamics (e.g., Whitehead and Luther, 1975; Olson and Singer, 1985; Griffiths, 1986; Campbell and Griffiths, 1990). Indeed, these images have been so influential that they now constitute a sort of ‘standard model’ of a mantle plume as a large, bulbous cavity (the head) trailed by a narrow conduit (the tail) connecting it with its source (Figure 5). According to this model, the arrival of a plume head at the base of the lithosphere produces massive flood basalts, while the trailing conduit generates the subsequent volcanic track (Richards et al., 1989). The model successfully explains important features of several prominent hot spots, including the volume of the topographic swell (e.g., Davies, 1988; Olson, 1990; Sleep, 1990) and the volume ratio between flood basalts and island chain volcanism (Olson and Singer, 1985; Richards et al., 1989), as discussed in more detail in Chapter 7.09.

7.03.8.1 Compositonally Buoyant Plumes

The early experiments on starting plumes in viscous fluids (Whitehead and Luther, 1975; Olson and Singer, 1985) were done by injecting compositionally buoyant fluid (typically corn syrup) at a constant rate from a pipe at the bottom of a large reservoir of a second fluid. The great advantage of this experimental setup is its simplicity: experiments can be performed under ambient laboratory conditions, with no temperature control required. Whitehead and Luther (1975) investigated the evolution of starting plumes for short times after the beginning of injection, during which the buoyant fluid forms a quasi-spherical ball that grows until it is large enough to ‘lift off’ from the injector (Figure 5). They proposed that the ball lifts off when its buoyant ascent speed (as predicted by Stokes law; see Chapter 7.04) exceeds the rate of increase of its radius, which occurs at a critical time $t_{sep}$ and radius $a_{sep}$ given by

$$a_{sep} = \left( \frac{4\pi}{3Q} \right)^{1/3} \left( \frac{\nu_m}{g} \right)^{1/4}, \quad t_{sep} = \left( \frac{3Q}{4\pi} \right)^{1/3} a_{sep}^{1/3} \tag{17}$$

where $Q$ is the volumetric injection rate, $\nu_m$ is the viscosity of the ambient fluid, $g^* = g (\rho_m - \rho_p)/\rho = g \Delta \rho/\rho$ is the reduced gravitational acceleration, and $\rho_m$ and $\rho_p$ are the densities of the ambient and injected fluids, respectively. Both the time and radius of separation increase with increasing viscosity of the ambient fluid. For the large viscosities typical of the mantle, plume heads must be quite large (radius $> 100$ km) to separate from their source (Whitehead and Luther, 1975).

After lift-off, the morphology and dynamics of starting plumes depend strongly on the viscosity ratio $\gamma = \nu_p/\nu_m$, where $\nu_p$ is the viscosity of the injected fluid (Whitehead and Luther, 1975; Olson and Singer, 1985). When the injected fluid is more viscous ($\gamma > 1$), the plume has roughly the shape of a cylinder (Figure 5(a)) whose length and radius increase with time. Olson and Singer (1985) called this a ‘diapiric’ plume. In the more geophysically realistic limit $\gamma << 1$, the ascending plume is a ‘cavity plume’ comprising a large head and a thin trailing conduit (Figure 5(b)). The rate of change of the volume $V$ of the head is just the injected flux $Q$ less the flux required to build the (lengthening) stem, or

$$\frac{dV}{dt} = Q - \pi R^2_1 W \tag{18}$$

where $R_1 \equiv (8\nu_p Q/\pi g^*)^{1/4}$ is the radius of the conduit required to carry the flux $Q$ by Poiseuille (cylindrical pipe) flow and $W$ is the velocity of the head predicted by Stokes’s law. For long times, a steady state ($dV/dt = 0$) is achieved in which the head reaches a terminal velocity $W_{term}$ and radius $a_{term}$ (Whitehead and Luther, 1975):

$$W_{term} = \left( g^* Q / 8\pi \nu_p \right)^{1/2}, \quad a_{term} = \left( 3W_{term} \nu_m / g^* \right)^{1/2} \tag{19}$$

More recent experiments have shown that under certain conditions compositionally buoyant cavity plumes can entrain the denser ambient fluid through which they rise (Neavel and Johnson, 1991; Kumagai, 2002). Kumagai (2002) identified two distinct regimes: a ‘vortex ring’ regime for $\gamma \leq 1$ in which layers of entrained fluid form a scroll-like pattern inside the plume head, and ‘chaotic stirring’ regime for $\gamma > 100$.
(see Section 7.03.10). Structures of the vortex ring type also occur in thermal starting plumes (Shlien, 1976; Shlien and Boxman, 1981; Tanny and Shlien, 1985; Griffiths and Campbell, 1990; Figure 7(c)).

The stem of a chemically buoyant plume can also support solitary waves, waves of large amplitude that propagate upwards without change in shape. Their existence was first demonstrated experimentally by Olson and Christensen (1986) and Scott et al. (1986), who generated the waves by increasing impulsively the volume flux $Q$ injected at the bottom of a vertical conduit surrounded by a fluid with a much greater viscosity. Whitehead and Helfrich (1988) subsequently showed that the flow within such waves exhibits recirculation along closed streamlines when viewed in a frame traveling with the wave, and suggested that this might provide a mechanism for transporting deep material rapidly to the surface with little contamination. Subsequent numerical experiments (Schubert et al., 1989) showed that thermal plume stems could also support solitary waves, and Laudenbach and Christensen (2001; Figure 41) performed an experimental study of solitary waves of hot fluid in a vertical conduit. Solitary waves on the stems of thermal plumes have been invoked to explain episodic magma production at weak hot spots and surges of activity at stronger hot spots (Laudenbach and Christensen, 2001) and the formation of the V-shaped ridges on the Reykjanes ridge south of Iceland (Albers and Christensen, 2001; Ito, 2001).

### 7.03.8.2 Thermal Plumes

Because the buoyancy force that causes plumes to rise in the Earth’s mantle is, to a large degree, thermal in origin, numerous laboratory investigations of thermal plumes in viscous fluids have been carried out since the mid-1980s. The first experiments on laminar starting plumes (Shlien and Thompson, 1975; Shlien, 1976) were not motivated by geophysical applications, and were performed using water and a narrow electrode as a heater. Experiments in this configuration typically used interferometry and particle tracking to recover the temperature (Figure 12(c)) and velocity fields (e.g., Shlien and Boxman, 1979; Tanny and Shlien, 1985; Chay and Shlien, 1986). By contrast, investigations of plumes in more viscous fluids have until now mostly focused on more global features such as the volume and ascent rate of the plume head. Because the viscosity of many experimental fluids decreases strongly with increasing temperature, hot thermal starting plumes generally are cavity plumes comprising a large head and a narrow stem. In some experiments, however, the heat source is suddenly turned off to produce an isolated head (‘thermal’) that rises without a trailing conduit. The discussion below begins with this latter case (Figure 6(a)), and then turns to the dynamics of steady plume stems and thermal starting plumes (Figures 42 and 43).

#### 7.03.8.2.1 Thermals

Griffiths (1986) conducted a laboratory study of thermals generated by injecting a known volume of heated polybutene oil into a large tank of the same oil at room temperature. He observed that a thermal initially rises at constant speed through a distance of the order of its diameter, and then slows down while increasing in volume. Griffiths proposed that the enlargement occurs because diffusion of heat away from the thermal warms a thin boundary layer of ambient fluid around it, which is then advected back to the thermal’s trailing edge and entrained into it. The total buoyancy of the thermal is therefore constant, and is proportional to the Rayleigh number $Ra = \alpha g \Delta T V_0 / \kappa \nu_m$, where $V_0 = \pi a_0^3$ is the thermal’s
initial volume and $\Delta T_0$ its initial temperature excess. Griffiths (1986) suggested that for times greater than $\tau_C \sim Ra^{-1/2} a_0^2 / \kappa$, the thermal’s volume $V$ and velocity $W$ have the self-similar forms

$$V = \frac{\pi C_1^3}{6} Ra^{3/4} (\kappa T)^{3/2}, \quad W = \frac{f}{2 \pi C_1} Ra^{3/4} (\kappa T)^{-1/2} \quad [20]$$

where $f = O(1)$ is a function of the viscosity contrast between the thermal and the ambient fluid and $C_1 \sim 1.0$ is a constant determined experimentally. Laboratory experiments using viscous oils (Griffiths, 1986a) and corn syrup (Coulliette and Loper, 1995) are consistent with [20]. In the mantle, thermals with radii $> 350$ km would reach the lithosphere before being influenced by thermal diffusion in typically 20 My and their ascent speeds could reach 10–20 cm yr$^{-1}$.

### 7.03.8.2.2 The stem

When a starting plume rises from a steady point source of heat, the lower part of its stem quickly achieves a steady-state structure that is independent of the dynamics of the ascending head above. To understand this structure, Batchelor (1954) studied analytically a model in which a plume rises from a point source of heat of strength $P$ in an infinite fluid with constant viscosity $\nu$, thermal diffusivity $\kappa$, specific heat $C_p$, and thermal expansion coefficient $\alpha$. Simple scaling arguments suggest that the vertical velocity $W$, the temperature anomaly $\Delta T$, and the stem radius $R$ scale with the height $z$ above the source as

$$W \sim \left( \frac{g \alpha P}{\pi \rho C_p \nu} \right)^{1/2} z^{1/2}, \quad \Delta T \sim \frac{P}{\kappa \rho C_p z} \quad [21]$$

$$R \sim \left( \frac{\rho \alpha C_p \kappa}{g \alpha P} \right)^{1/4} z^{1/2}$$
The vertical velocity is constant, but thermal diffusion causes the temperature anomaly to decrease with height as \( z^{-1} \) and the stem radius to increase as \( z^{1/2} \). The scalings \([21]\) are valid to within multiplicative functions of the Prandtl number \( Pr = \nu/\kappa \), which have been determined numerically and/or analytically (Fuji, 1963; Worster, 1986; Vasquez et al., 1996). Experiments in water (\( Pr \sim 7 \)) verify the scalings \([22]\) and agree with the complete functional form of Fuji (1963) if corrections are made for the finite size of the heat source and for downward heat losses from it (Shlien and Boxman, 1979; Tanny and Shlien, 1985).

The scaling \([21]\) is no longer valid in the limit of infinite \( Pr \), because the model problem studied by Batchelor (1954) has no solution if the fluid is infinite and inertia is totally absent (Stokes paradox; see Chapter 7.07). However, a solution does exist if the heat source is located on a horizontal (rigid or free) boundary, and has a vertical velocity \( W \) given by \([21]\) multiplied by a function that increases logarithmically with height (Whittaker and Lister, 2006).

The dynamics are very different when the viscosity depends strongly on temperature, as is the case in the mantle. Figure 43 shows isotherms \((a-c)\) and profiles of vertical velocity on the axis \((d)\) at different times for a thermal plume above a heater in corn syrup (Vatteville, 2004). The lower portion 20 mm \( \leq z \leq 70 \) mm of the plume quickly reaches a steady state, as shown by the similarity of the isotherms and velocity profiles for \( t = 120 \) s and 300 s in this height range. Moreover, the vertical velocity decreases strongly with height, in contrast with the constant-viscosity case. Similar results were found by Laudenbach (2001) along the stem of a plume of heated corn syrup, which was injected at the bottom of a tank filled with the same fluid at room temperature.

These observations are well explained by Olson et al.’s (1993) approximate solution of the boundary-layer equations for the steady flow above a point source of heat in a fluid whose viscosity depends exponentially on temperature. The upwelling velocity \( W \) on the axis and the temperature anomaly \( \Delta T \) vary with height \( z \) as

\[
W = \left( \frac{g2 \rho P}{4\pi \rho C_v \nu_0(z)} \right)^{1/2} \Delta T = \Delta T_0 \exp \left( -\frac{4\pi k \Delta T_0 z}{P} \right) \tag{22}
\]

where \( \nu_0(z) \) is the viscosity on the plume centerline, \( \Delta T_0 \) is the temperature anomaly at the base of the plume, \( k \) is the thermal conductivity, and \( \Delta T_c \) is the temperature required to change the viscosity by a factor \( e \). The expression \([22]\) for \( W \) is identical to the constant-viscosity expression \([21]\) except for the variability of the centerline viscosity \( \nu_0(z) \). The temperature anomaly decreases exponentially with height, and the corresponding increase in viscosity causes the upwelling velocity \( W \) also to decrease with height, in qualitative agreement with the experimental observations. The differences relative to the constant-viscosity case are due to the fact that temperature-dependent viscosity concentrates the upwelling in the hottest central part of the plume stem, so that the radius \( r_c \) of the thermal anomaly is wider than the radius \( r_w \) of the upwelling region (the ‘conduit’ proper) even for \( Pr \gg 1 \) (Figure 42).

For typical mantle parameter values, plumes with large viscosity variations (\( \geq 1000 \)) would have \( \sigma_w \sim 25-30 \) km and \( \sigma_T \sim 60 \) km, while plumes with low viscosity variations (\( \leq 100 \)) are typically twice as broad.

### 7.03.8.2.3 Thermal starting plumes

A thermal starting plume is essentially an ascending thermal connected by a stem to a source of buoyancy (Figure 5). Thus, the plume head can grow both by entrainment of ambient fluid and by the addition of plume fluid through the stem. Unfortunately, no reliable scaling laws have yet been found to describe the evolution of the size and speed of the plume head in this case, and different studies give conflicting results.

Griffiths and Campbell (1990) investigated thermal starting plumes by injecting hot glucose syrup with a temperature excess \( \Delta T_0 \) into a cooler reservoir of the same fluid at a constant volumetric rate \( Q \). They proposed that the volume \( V \) of the plume head evolves as

\[
\frac{dV}{dt} = Q + C_2(\nu_0 \Delta T_0 Q t / \nu_m)^{1/2} \tag{23}
\]

where \( \nu_m \) is the viscosity of the cold ambient fluid and \( C_2 \) is an empirical entrainment constant. For large times the second (entrainment) term in \([23]\) is dominant, implying

\[
V = \left( \frac{4C_2}{9} \right)^{3/2} \beta \kappa^{3/4} e^{1/4} \tag{24}
\]
where \( \beta = (2g\Delta T Q / \nu_m)^{3/4} \). The corresponding velocity of the head, calculated from Stokes's law for a relatively inviscid drop, is

\[
W = \frac{1}{8} \frac{(6)}{(\pi)} \frac{2}{3} \frac{1}{2} \frac{3}{2} \frac{1}{4} C_z \beta(\nu_t)^{1/4} [25]
\]

On the basis of experiments with thermals, Griffiths and Campbell (1990) state that \( 1 \leq C_z \leq 4 \). Equation \([24]\) describes well the experimental data for intermediate times, although at larger times the cap growth rate becomes lower than predicted and is better described by the law \( V_t \sim t^{3/2} \) \([20]\) for an isolated thermal (Figure 44).

Experiments on plumes produced by localized heating without fluid injection (Moses et al., 1993; Coulliette and Loper, 1995; Kaminski and Jaupart, 2003; Vatteville, 2004) show a different behavior: the velocity of the head increases rapidly for a short time and then attains a nearly constant value \( W_{\text{term}} \). To explain this, Coulliette and Loper (1995) proposed a modified version of the Griffiths and Campbell (1990) model that accounted for incomplete entrainment of the hot thermal boundary layer surrounding the head. However, the resulting theory involves empirical constants whose values vary by factors of 2–5 among experiments. A possible cause of the difficulty may be the theory’s neglect of the flux required to build the lengthening stem, which is important for compositional plumes (Whitehead and Luther, 1975; Olson and Singer, 1985).

### 7.03.8.3 Interaction of Plumes with a Large-Scale Flow

Because deep-seated plumes must traverse the convecting mantle on their way to the surface, it is important to investigate how they interact with an ambient large-scale flow. This interaction was first studied by Skilbeck and Whitehead (1978), who injected compositionally buoyant fluid into a shear flow generated by rotating the top surface of a tank containing another fluid. As the rising plume head is swept horizontally away from the source, the trailing conduit is tilted until it becomes gravitationally unstable at a tilt angle \( \approx 30^\circ \) from the vertical. The instability forms new, smaller cavity plumes whose spacing is independent of the volume flux \( Q \) (Whitehead, 1982). Olson and Singer (1985) performed similar experiments by towing the source at constant speed \( U \) at the bottom of a tank of motionless fluid. The characteristic spacing \( \gamma \) and volume \( V \) of the new cavity plumes generated by the instability were found to scale as \( \lambda \approx 12(\nu_m U / g^2)^{1/2} \) and \( V \approx 12(\nu_m / g^2 U)^{1/2} Q \), respectively, although these relationships break down for small (<0.3) and large (>10) values of the dimensionless parameter \( (gQ / \nu_m U^2)^{1/2} \). Skilbeck and Whitehead (1978) suggested that this instability might account for the discrete character of volcanic island chains.

Richards and Griffiths (1988) studied experimentally the interaction between a compositionally buoyant plume conduit and a large-scale shear flow, for low conduit tilt angles at which the instability documented by Skilbeck and Whitehead (1978) does not occur. They showed that the conduit’s shape can be calculated by describing its motion as a vector sum of the shear velocity and a modified vertical Stokes velocity \( \nu_s \) for individual conduit elements. For a simple linear shear flow, the resulting steady-state shape is a parabola. In response to a sudden change in the shear flow, the lateral position \( x(z) \) of such a conduit adjusts to a new steady-state position in a time \( z / \nu_s \), where \( z \) is the height. Griffiths and Richards (1989) applied this theory to Hawaii, concluding that the sharpness of the bend in the Hawaiian–Emperor chain at 43 Ma implies that the deflection of the Hawaiian plume before the change in plate motion was <200 km.

Kerr and Mériaux (2004) carried out a comprehensive laboratory study of thermal plumes ascending in a shear flow in a fluid with temperature-dependent viscosity. Plumes were observed to rise initially with a constant velocity that is

---

**Figure 44** Volume of a starting plume as a function of time. The circles represent the experimental data points from Griffiths and Campbell (1990). In dashed lines, the \( r^{1/3} \) law for a growing Stokes sphere (Whitehead and Luther, 1975), the \( r^{6/4} \) growth model of Griffiths and Campbell (1990), and the \( r^{3/2} \) law due to the conductive growth of the plume cap.
independent of the centerline viscosity $\nu_p$. Subsequently, the ambient shear tilts the plume conduits, and in some cases significant cross-stream circulation and entrainment is seen. However, the tilted conduits never develop Rayleigh–Taylor-type gravitational instabilities of the kind observed on tilted compositional conduits, perhaps because thermal diffusion is fast enough to suppress their growth (Richards and Griffiths, 1988).

### 7.03.8.4 Plume–Lithosphere Interactions

Because the interaction of plumes with the lithosphere is responsible for most of the geochemical and geophysical signatures observed at hot spots, the dynamics of this process has been the focus of numerous experimental studies. The more general case of the interaction of plumes with density and/or viscosity boundaries within the mantle will be treated together with two-layer convection in Section 7.03.9.

#### 7.03.8.4.1 Interaction with a stagnant lithosphere

To understand the formation of bathymetric swells by plumes, Olson and Nam (1986) performed laboratory experiments on the deformation of an initially spherical drop of buoyant low-viscosity fluid rising toward a high-viscosity boundary layer at the cooled free surface of a fluid with strongly temperature-dependent viscosity. They measured the amplitude of the surface topography as a function of time using a proximity probe (Section 7.03.3.5; Figure 17), and found that the typical evolution comprises a phase of rapid uplift followed by slower subsidence that corresponds to stagnation and lateral spreading of the drop below the cold boundary layer. The maximum swell height increased with the drop volume, but decreased with increasing lithosphere thickness. Griffiths et al. (1989) confirmed these results using holographic interferometry (Figure 17), and extended the study to larger viscosity and buoyancy contrasts between the lithosphere and the underlying mantle. They found the maximum swell height to be independent of the viscosity contrast, but a decreasing function of the density contrast.

Using experiments performed under isothermal conditions, Griffiths and Campbell (1991) determined scaling laws for the radius of a drop spreading beneath a solid plate and the thickness of the ‘squeeze film’ above the drop as functions of time. They observed that the gravitationally unstable interface between the drop and the squeeze film eventually broke up via a Rayleigh–Taylor instability with a complex lateral planform dominated by short-wavelength (~drop thickness) upwellings and downwellings.

Olson et al. (1988) studied the penetration of mantle plumes through colder and more viscous lithosphere, whereas Jurine et al. (2005) investigated how thermal plumes deform and penetrate a compositionally distinct lithosphere. The amplitude and shape of the penetration was found to depend mostly on the ratio $B$ between the compositional and thermal density contrasts (buoyancy number; see Section 7.03.9), while the timescale of the interface displacement is governed primarily by the viscosity contrast. When applied to the continental lithosphere, the experimentally derived scaling laws show that plume penetration through Archean lithosphere requires thermal anomalies larger than 300 K, corresponding to buoyancy fluxes in excess of those determined today for the Hawaiian plume. But because the experimental plumes are observed to interact differently with lithospheres of different ages and/or compositions, no single ‘typical’ plume signature can be identified.

#### 7.03.8.4.2 Interaction with a moving or rifting lithosphere

While the interaction of a plume with a stationary lithosphere is relatively easy to study in the laboratory, analog modeling of a moving lithosphere poses serious challenges, including the difficulty of controlling the temperature on the moving boundary and the unwanted influence of the return flow the boundary motion induces (see Section 7.03.7). Here two cases must be distinguished: interaction of the plume with an intact lithosphere moving at constant speed relative to it (‘plume–plate interaction’ or PPI), and interaction of the plume with a rifting lithosphere representing a mid-ocean ridge (‘plume–ridge interaction’ or PRI). To date there exist no experimental studies of PPI, which has been investigated solely with analytical and numerical methods (e.g., Olson, 1990; Ribe and Christensen, 1994, 1999). By contrast, PRI has been successfully modeled in the laboratory.

The first case to be studied was that of a ‘ridge-centered’ plume rising directly beneath a spreading ridge, Iceland being the main natural example. Feighner and Richards (1995) investigated this case by injecting compositionally buoyant fluid from a pipe beneath the boundary between two diverging
mylar sheets at the top of a tank of corn syrup. They observed that the buoyant fluid spreads beneath the mylar sheets to form a thin layer with a steady 'bow-tie' shape in mapview. They found that the width of the layer along the ridge scales as \((Q/U)^{1/2}\), where \(Q\) is the volumetric injection rate and \(U\) is the half-spooling rate, in agreement with the predictions of numerical models based on lubrication theory (Ribe et al., 1995) and the tracking of chemically buoyant tracer particles (Feighner et al., 1995). This work was extended by Kincaid et al. (1995) to the interaction of a thermal plume with a mylar-sheet spreading ridge located at some distance from it. The mantle was modeled using a concentrated sucrose solution, producing a highly viscous upper boundary layer (the ‘lithosphere’) that cooled and thickened as it moved away from the ridge. Kincaid et al. (1995) observed that the plume material can be guided toward the ridge along the sloping base of the lithosphere, confirming the suggestion of Schilling (1991) that an off-axis plume may communicate thermally and chemically with a spreading ridge.

### 7.03.9 Inhomogeneous Fluids: Mixing and Thermochemical Convection

These studies have arisen from two fundamental questions in mantle dynamics:

1. How can the dispersion observed in geochemical data be interpreted in dynamical terms? Since geochemical isotopic signatures modify neither the density nor the physical properties (rheology, thermal expansion, and thermal diffusion) of mantle materials, they constitute passive tracers of mantle flow (e.g., Allègre, 1987). The question therefore relates to the more general problem of convective mixing of passive tracers.

2. What is the convective pattern in the mantle? Can mantle convection be stratified? There are numerous sources of compositional heterogeneities in the mantle, such as slab remnants (e.g., Olson and Kincaid, 1991; Christensen and Hofmann, 1994), delaminated continental material, relics of a primitive mantle (Gurnis and Davies, 1986; Tackley, 1998; Kellogg et al., 1999) enriched, for example, in iron (Javoy 1999), or chemical reactions with or infiltration from the core (e.g., Hansen and Yuen, 1988). So, how does a density heterogeneity interact with thermal convection?

Both questions involve time-dependent 3-D phenomena occurring on a wide range of spatial scales, abrupt gradients of physical properties (e.g., density and viscosity), and intricate physical phenomena (stirring, entrainment) for which even the equations remain to be found. Laboratory experiments have therefore proved to be a good tool to tackle these problems. A more complete review of mixing in the mantle is given by Tackley in Chapter 7.10.

#### 7.03.9.1 Mixing of Passive Tracers

An impurity is said to be a passive tracer when it does not modify the flow field transporting it. This condition requires the impurity either to be neutrally buoyant, or to have a characteristic length scale very small compared to that of the flow. Mixing of an impurity involves two phenomena: (1) stirring, that is, stretching and folding of the heterogeneity until its length scale becomes small enough for chemical diffusion to act, followed by (2) diffusive erasure of the heterogeneity (e.g., Reynolds, 1894; Nagata, 1975; Ottino, 1989). Given the small values of chemical diffusion coefficients in the mantle (typically \(10^{-15}\) \(\text{cm}^2\,\text{s}^{-1}\); Hofmann and Hart, 1978), the latter process will act only over a few centimeters on geological timescales. Stirring is therefore the process of greater interest for geodynamics (see Chapters 7.02 and 7.10).

Our current understanding of mixing has significantly improved in the last 20 years, due to the establishment of a clear connection between chaos and stirring (e.g., Aref, 1984). Essential to this breakthrough were careful laboratory experiments in prototypical flows, first in 2-D, time periodic flows (e.g., Ottino et al., 1988) then in 3-D flows (e.g., Fountain et al., 1998). In particular, studies in simple periodic 2-D flows have helped visualize chaos (Figure 45): unmixed regions translate, stretch, and contract periodically, and represent the primary obstacle to efficient mixing. Particle trajectories in chaotic regions separate exponentially in time, and material filaments are continuously stretched and folded by means of horseshoes. The details seen in Figure 45 would have been impossible to see in computer simulations since numerical diffusion is greater than the molecular diffusion in the fluid experiments. However, a special effort has been made in mixing studies to couple to the extent possible simple lab experiments and numerical simulations of the same flow, since the latter give much more ready access to the velocity field and
other quantities such as the time-dependent stretching field. Recent theoretical work has provided a solid connection between maxima of the stretching field and invariant manifolds of the flow (Haller and Yang, 2000; Haller, 2001). In a flow which can be described analytically, the comparison between computations of stretching and dye visualization shows that the dye spreads over regions that have experienced large stretching. The use of high-resolution PIV technique in laboratory experiments now allows us to calculate the in situ stretching field (e.g., Voth et al., 2002). This gives a powerful new insight into the geometrical structure which underlies mixing, even for nonanalytical flow field.

The more specific study of mixing by thermal convection in the laboratory has been confined to 2-D and weakly 3-D patterns. Solomon and Gollub (1988a, 1988b) studied the passive transport of methylene blue and latex spheres by steady 2-D Rayleigh–Benard convection. The convective flow was monitored by laser Doppler velocimetry and the transport by optical absorption techniques. The transport in this system is determined by the interplay between advection of tracer particles along the streamlines within convection rolls and diffusion of these particles between adjacent rolls. They found that the transport on long space and timescales could be modeled as a diffusive process with an effective diffusion coefficient $D^* \approx D Pe^{1/2} = D (W d/\pi D)^{1/2}$, where $D$ is the molecular diffusion, $W$ the characteristic velocity of the flow, and $Pe$ the Peclet number.

When the 2-D convection is time periodic, the local effective diffusion coefficient becomes independent of the molecular diffusion but depends linearly on the local amplitude of the roll oscillation. The basic mechanism of transport is chaotic advection in the vicinity of oscillating roll boundaries (Solomon and Gollub, 1988b). Solomon et al. (1996) verified that $D^*$ is related to the areas of the lobes which are carrying tracers between two cells (Figure 46). For weakly time-dependent and 3-D flows, Solomon and Mezic (2003) observed completely uniform mixing when the pattern oscillation period was close to typical circulation times, as predicted by ‘singularity-diffusion’ theory.

### 7.03.9.2 Convection in an Initially Stratified Fluid

The most commonly studied situation comprises two superposed fluid layers of different composition, density, and viscosity (Figure 47) across which a temperature difference $\Delta T$ is applied. Since convection occurs in the Earth’s mantle on large scales ($\gg 1 \text{ km}$), and surface tension acts on millimeter scales, we shall confine our review to the case of two miscible fluids. When the viscosity depends on temperature, $\nu_m$ and $\nu_d$ are taken to be the values at

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**Figure 45** Mixing produced by discontinuous cavity flow. The top wall moves from left to right for half a period and then stops, then the bottom wall moves from right to left and stops, and so on. Visualization is provided by a fluorescent tracer dissolved in glycerine excited by a longwave ultraviolet light of 365 nm. Folding and an island of unmixed material are clearly seen. From Ottino JM, Leong CW, Rising H, and Swanson PD (1988) Morphological structures produced by mixing in chaotic flows. *Nature* 333: 419–426.

the mean temperature of each layer. In cases where the thermal expansion coefficient also depends on temperature (Le Bars and Davaille, 2004a, 2004b), it is taken to be the value at the temperature of the layer interface. Accordingly, the system dynamics are characterized by five dimensionless numbers:

- the Rayleigh number, \( Ra(H, \Delta T) \) defined using eqn [1] with \( \nu = \max(\nu_k, \nu_m) \);
- the Prandtl number \( Pr = \nu/\kappa \);
- the viscosity ratio \( \gamma \);
- the depth ratio \( a = b_d/H_k \) and
- the buoyancy ratio, \( B = \Delta \rho_d/\rho \Delta T_f \), ratio of the stabilizing chemical density anomaly \( \Delta \rho \) to the destabilizing thermal density anomaly.

### 7.03.9.2.1 Regime diagrams

Early work used linear stability analysis to investigate the stability of various two-layer systems as a function of Rayleigh number and density contrast (Richter and Johnson, 1974; Busse, 1981; Sotin and Parmentier, 1989). The presence of a deformable interface offers the possibility of Hopf bifurcations: since it introduces an additional degree of freedom, overstability can occur in the form of oscillatory interfacial instability (Richter and Johnson, 1974; Renard and Joseph, 1985; Renardy and Renardy, 1985; Rasenat et al., 1989; Le Bars and Davaille, 2002; Jaupart et al., 2007). This overstable mode was observed experimentally for the first time by Le Bars and Davaille (2002) (Figure 48). At first, numerical studies of two-layer convection at finite amplitude have typically focused on the mechanism of coupling (thermal vs. mechanical) between the layers, often treating the interface as impermeable (e.g., Richter and McKenzie, 1981; Kenyon and Turcotte, 1983; Christensen and Yuen, 1984; Cserespes and Rabinowicz, 1985; Boss and Sacks, 1986; Cserespes et al., 1988). For a long time, the highly nonlinear convective regimes associated with high Rayleigh numbers or entrainment at the interface were accessible only to laboratory experiments.

Using aqueous glycerol solutions, Richter and McKenzie (1981) showed that two-layer convection is stable when the ratio of chemical to thermal buoyancy \( B \) exceeds unity, but they did not address the issue of partial mixing. Olson (1984) used aqueous sugar solutions to study two-layer convection in the penetrative regime \( B \approx 1 \) when the viscosity ratio between the two layers is lower than 10. His results

![Figure 47](image1.png) Sketch of two-layer convection. The denser layer is initially at the bottom.

![Figure 48](image2.png) (a) Marginal stability curves separating the different convective regimes. The curves here were calculated for the case \( \gamma = 6.7 \) and \( a = 0.5 \), where \( Ra_0 = 5430 \), \( Ra_c = 38227 \) and \( B_c = 0.302 \). For \( B < B_c \), the whole-layer regime develops under the form of (b) overturn for \( \gamma \) around 1 and/or low \( B \) (here \( B = 0.048; Ra = 6.7 \times 10^3; a = 0.44; \gamma = 1.1 \)), or (c) traveling waves at low Rayleigh number (here \( B = 0.20; Ra = 1.8 \times 10^4; a = 0.5; \gamma = 6.7 \)). From Le Bars M and Davaille A (2002) Stability of thermal convection in two superimposed miscible viscous fluids. Journal of Fluid Mechanics 471: 339–363.
established that entrainment at low Reynolds number can indeed occur, and that viscous stresses can substitute for inertial instabilities as a mixing mechanism. Nataf et al. (1988) demonstrated that the mechanism of coupling between immiscible fluid layers depends on conditions at the interface (e.g., surface tension and impurities), and that the oscillatory interfacial mode is stabilized by surface tension, as predicted by theory.

Numerous studies now exist on the subject and the domain of parameters investigated is therefore large, with $10^3 < Ra < 10^9$, $10^{-2} < \gamma < 6 \times 10^3$, $0.05 < a < 0.95$, $0.04 < B < 10$. The experiments have been run with constant viscosity (e.g., aqueous Natrosol solutions) or temperature-dependent viscosity fluids, (e.g., sugar or corn syrups) and with constant or temperature-dependent thermal expansion. The different regimes observed in laboratory experiments (Richter and McKenzie, 1981; Olson, 1984; Olson and Kincaid, 1991; Davaille, 1999a, 1999b; Davaille et al., 2002; Le Bars and Davaille, 2002, 2004a; Jellinek and Manga, 2002, 2004; Cottrell et al., 2004; Jaupart et al., 2007), but also in numerical simulations (Schmeling, 1988; Tackley, 1998, 2002; Kellogg et al., 1999; Montague and Kellogg, 2000; Hansen and Yuen, 2000; Samuel and Farnetani, 2002; McNamara and Zhong, 2004) are shown in Figure 49 and Table 1 and described in more detail below.

For $B < 0.03$, chemical density heterogeneities are negligible and the system behaves as a single homogeneously fluid (Figure 49(a)). For larger values of $B$ and $Ra > 10^5$, two scales of convection coexist: compositionally homogeneous thermal plumes generated at the outer boundaries, and large-scale thermochemical instabilities that involve both fluids.

**Whole-layer regime and unstable doming.** When $B$ is less than a critical value $B_c \sim 0.4$ (the exact value depending on the viscosity and depth ratios; Le Bars and Davaille, 2002; Jaupart et al., 2007), the stable compositional stratification can be overcome by thermal buoyancy. The interface between the layers then becomes unstable, and convection occurs over the whole depth. An important characteristic of the interfacial instability is the ‘spouting’ direction (Whitehead and Luther, 1975), defined as the direction (up or down) in which finite-amplitude perturbations grow superexponentially to form dome-like structures (Figures 49(e) and 49(f)). The theory of the Rayleigh–Taylor instability shows that spouting occurs in the direction of the lower layer with the lower ‘resistance’ (Ribe, 1998). Thus, the lower layer will spout into the upper one only if

$$b_t < \frac{H}{1 + \gamma^{-1/3}}$$  \[26\]

otherwise, spouting is downward (Le Bars and Davaille, 2004a). Condition [26] implies, for example, that a less viscous layer will spout into a more viscous overlying mantle only if the former is much thinner. As for the case of purely compositional plumes (Figures 5 and 18), the morphology of thermochemical instabilities depends on the viscosity ratio $\gamma$. Suppose for definiteness that [26] is satisfied, so that spouting is upward. Then when the lower layer is more viscous ($\gamma > 1$), large cylindrical diapirs form (Figure 49(f)), and secondary plumes can develop above them in the upper layer (Davaille, 1999b). When $\gamma < 1$, by contrast, purely thermal instabilities first develop within the lower layer, and then merge to form large cavity plume heads (Figure 49(e)). In both cases, if the lower layer is thin, the plumes empty it before they reach the upper boundary, and become disconnected from the lower boundary as in the purely thermal case (Figure 26). If the layer is thicker and/or $0.2 < B < B_c$, the plumes reach the upper boundary before disconnecting from the lower (Figure 49(e)). They then begin to cool and lose their thermal buoyancy, and eventually collapse back to the bottom, whereupon the cycle begins again. When $\gamma > 5$ or $\gamma < 0.2$, each layer retains its identity over several pulsations. Since thermal buoyancy must overcome the stable compositional stratification before driving convection, the temperature anomaly $\theta$ carried by thermochemical instabilities is the slave of the compositional field (Le Bars and Davaille, 2004a) and is

$$\theta = (0.98 \pm 0.12) \frac{\Delta \rho_x}{\alpha \rho}$$  \[27\]

Moreover, the diameter, wavelength, velocity, and cyclicity of the domes are mainly controlled by the more viscous upper layer since it retards motion over the whole depth (e.g., Whitehead and Luther, 1975; Olson and Singer, 1985; Herrick and Parmentier, 1994).

**Stratified regime: anchored hot spots, bumps, ridges, and piles.** For $B > B_c$, thermal buoyancy cannot overcome the stable compositional stratification. Convection remains stratified, and a TBL forms at the interface from which long-lived thermochemical plumes are
These plumes do not have a well-defined head, and the thermal anomaly they carry is weak, proportional to the temperature difference across the unstable part of the TBL above the interface (Christensen, 1984; Farnetani, 1997; Tackley, 1998). They entrain a thin filament (at most 5% of the total plume volume) of the denser bottom layer by viscous coupling and locally deform
Table 1  Convective regime and upwellings morphology as a function of $B$, viscosity ratio $\gamma$, layer depth ratio $a$, and internal Rayleigh number of the denser bottom layer $Ra_d$

<table>
<thead>
<tr>
<th>$B$</th>
<th>Regime</th>
<th>$\gamma = \eta_d/\eta_m$</th>
<th>$Ra_d$</th>
<th>$a = h_d/H$</th>
<th>Upwellings morphology</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt;0.03$</td>
<td>1-layer</td>
<td></td>
<td></td>
<td></td>
<td>Thermal plumes</td>
<td>49a</td>
</tr>
<tr>
<td>$0.03 &lt; B &lt; B_c \sim 0.4$</td>
<td>Whole layer</td>
<td>$&lt;Ra_c \sim 1000$</td>
<td></td>
<td></td>
<td>Passive ridges = return flow to downwellings</td>
<td>49c</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$&lt;1$</td>
<td>$Ra_c$</td>
<td>$a &lt; a_c$</td>
<td>Active domes and passive ridges</td>
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<td></td>
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<td></td>
<td></td>
<td>Cavity plumes (or 'mega-plumes') through collection of small thermal instabilities</td>
<td>49e</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>detach from hot bottom boundary (HBB)</td>
<td>49g</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$&gt;Ra_c$</td>
<td></td>
<td>$a &gt; a_c$</td>
<td>Passive ridges</td>
<td>49c</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>return flow to cold more viscous downwellings</td>
<td></td>
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<tr>
<td>$&gt;1$</td>
<td>$&gt;Ra_c$</td>
<td></td>
<td></td>
<td></td>
<td>Active hot diapirs</td>
<td>49f</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>detach from HBB if $a &lt; 0.3$ and $B &lt; 0.2$</td>
<td>49g</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>continuous fingers from HBB otherwise</td>
<td>49h</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>Secondary plumes on top of domes</td>
<td></td>
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<tr>
<td>$1/5 &lt; \gamma &lt; 5$</td>
<td>Overturning = immediate stirring after first instabilities</td>
<td></td>
<td></td>
<td></td>
<td>Pulsations = two layers retain their identity for several doming cycles</td>
<td>49f + 49h</td>
</tr>
<tr>
<td>$\gamma &lt; 1/5$ or $\gamma &gt; 5$</td>
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</tbody>
</table>

For $B < B_c$, 2-layers

| $B > 1$   |          |                           |        |           | Nearly flat interface                                                                 | 49d    |
| $B_c < B < 1$ | Dynamic topography | $<Ra_c$      |        |           | Dynamic topography, does not reach the upper boundary                                 | 49b-c  |
|           | $<1$     |                           |        |           | Passive ridges (2D) or piles (3D) formed in response to cold viscous downwellings    | 49c    |
|           | $>1$     | $Ra_c < Ra_d < 10^4$     |        |           | Passive ridges (2D) or piles (3D)                                                    | 49c    |
|           |          | $Ra_d > 10^4$             |        |           | Upwelling domes, or superplumes                                                      | 49b    |

For depth- or temperature-dependent properties, the viscosities are taken at the averaged temperature of each layer, and $a$ is taken at the interface. We focus on high global $Ra (>10^6$).

Note that if $Ra_d < Ra_c$, the denser layer cannot convect on its own.
the interface into cusps (Figure 49(d); Davaille, 1999b; Jellinek and Manga, 2002, 2004). The interfacial topography and temperature anomaly serves in turn to anchor the plumes (Davaille 1999a, 1999b; Namiki and Kurita, 1999; Davaille et al., 2002; Jellinek and Manga, 2002, 2004; Matsumo et al., 2006), which persist until the chemical stratification disappears through entrainment.

When $B_c < B < 1$, thermal buoyancy is sufficient to maintain local thermochemical ‘bumps’ on the interface (Figures 49(b) and 49(c)), whose maximum height increases with increasing $Ra$ and decreasing $B$ (Le Bars and Davaille, 2004a; McNamara and Zhong, 2004; Cottrell et al., 2004; Jaupart et al., 2007). If the lower layer is thin enough, it can break up and form stable ‘ridges’ and ‘piles’ (Hansen and Yuen, 1988; Tackley 1998, 2002; Table 1). The maximum height of the thermochemical bumps is the height of ‘neutral’ buoyancy, where local compositional buoyancy is just balanced by local thermal buoyancy (LeBars and Davaille, 2004a; Kumagai et al., 2007). Upon reaching this level, the dome can, depending on its internal Rayleigh number, either stagnate or collapse. This generates very complicated morphologies (Figure 50).

Relevance of the experiments to the deep mantle: pressure dependence of $B$. The laboratory experiments span well the parameter space of the mantle, except in one respect: the depth dependence of the physical properties. This can be a problem since in the mantle, $B$ could vary with pressure (see Chapter 2.06), henceforth depth, because of two effects: (1) the thermal expansion coefficient $\alpha$ varies with depth (e.g., Tackley, 1998; Montague and Kellogg, 2000; Hansen and Yuen, 2000), and/or (2) the density contrast of compositional origin $\Delta \rho_X$ varies with depth since the two materials have different compressibilities (Tan and Gurnis, 2005; Samuel and Bercovici, 2006).

1. The interface stability is determined by the ability of the thermal density contrast across the interface to overcome the compositional density contrast. The former should therefore be calculated using the value of $\alpha$ at the interface depth, and we do not expect $B_c$ to vary significantly (see discussion in LeBars and Davaille (2004a)). This is indeed confirmed by Figure 48, where both the numerical simulation results using a depth-dependent $\alpha$ and laboratory experiments collapse on the same diagram. In the mantle, $\alpha$ decreases with increasing depth (see Chapter 2.06), which, for a given compositional density contrast, has two effects on thermochemical convection. First, the regime of a given two-layer system will change with the location of the interface (Davaille, 1999b), from doming in the mid-mantle to stratified just above the CMB. Second, any dome starting deep in the mantle will gain buoyancy as it rises, which might enhance its velocity (LeBars and Davaille, 2004b).

2. Now, Si-enriched (i.e., denser) compositions would also induce a lower compressibility of the dense anomaly compared to the surrounding mantle, implying an increase of the compositional density excess with decreasing depth (Samuel and Bercovici, 2006; Tan and Gurnis, 2005). Therefore, although Si-enriched thermochemical domes could fully develop and rise toward the surface if $B < B_c$ initially, upon reaching the level of neutral buoyancy they would stagnate and spread under it. If this level is significantly higher than the initial interface, large and well-formed plume heads could therefore stagnate at various depths of the lower mantle (Samuel and Bercovici, 2006). If the neutral buoyancy level is around the initial interface depth, tabular less viscous piles with steep vertical walls could form (Tan and Gurnis, 2005). This is the only way to generate less viscous active ridges or piles.

Stability of the continental lithosphere. Combining stability analysis and experiments, Cottrell et al. (2004) and Jaupart et al. (2007) studied the stability of the cooling continental lithosphere (see Chapters 7.07 and 6.05), a thin, compositionally lighter, but also more viscous, layer on top of an (quasi)-infinite mantle. For this combination of transient cooling from above,
small depth ratio \( a \) and large viscosity ratio \( \gamma \), two types of oscillatory instabilities are obtained, depending on the value of \( B \) (Jaupart et al., 2007). In particular, for \( 0.275 < B < \sim 0.5 \), oscillations are vertical and the lithosphere could grow by entrainment of chemically denser material from the asthenosphere. These studies further suggest that the lithosphere could have developed in a state near that of instability with different thicknesses depending on its intrinsic buoyancy.

**Diversity of mantle upwellings.** As described by Ito and van Keken in Chapter 7.09, hot spots and mantle upwellings present a large range of different signatures, which are difficult to reconcile with one single origin: there are probably different kinds of hot spots and upwellings in the mantle. According to Table 1 and Figure 49, this diversity could be explained by the coexistence of the different regimes of thermochemical convection in a compositionally heterogeneous mantle which convects today primarily on one layer (e.g., Davaille, 1999; Davaille et al., 2005). According to Section 7.03.7.3, these thermochemical upwellings would develop primarily away from downwellings (e.g., Gonnermann et al., 2004; McNamara and Zhong, 2004), that is, in two mantle ‘boxes’ separated by the circum-Pacific subduction zones (e.g., Davaille et al., 2002; McNamara and Zhong, 2005).

Then, long-lived hot spots could be due to plumes anchored on the topography of a dense \( (B > 0.5) \) basal layer, which could also develop piles and ridges. Scaling laws based on laboratory experiments (Davaille, 1999a, 1999b; Davaille et al., 2002; Jellinek and Manga, 2004) suggest that plumes anchored in \( D^c \) could develop for \( \Delta \rho_Y > 0.6\% \), and that they could survive hundreds of millions of years, depending on the spatial extent and magnitude of the anchoring chemical heterogeneity (Davaille et al., 2002; Gonnermann et al., 2002; Jellinek and Manga, 2002, 2004). Such plumes could therefore produce much longer-lived hot spots than the transient plumes observed experimentally in isochemical convection (Figure 27). However, plume and piles longevity does not necessarily imply spatial fixity: plumes and their anchors could be advected by large-scale flow associated with strong downwellings such as subducting plates (Tan et al., 2002; Davaille et al., 2002; McNamara and Zhong, 2004).

On the other hand, ‘supperswells’, hot spot clusters and large seismic slow anomalies in the deep mantle could be due to doming of a compositionally denser layer with \( B < 0.5 \). Then, according to [27], thermochemical instabilities with temperature anomalies \( 300–500 \) K could be produced at the base of the mantle by compositional density heterogeneities of \( 0.3–0.6\% \). If viscosity depends only on temperature, the instabilities would then be \( 10–3000 \) times less viscous than the bulk mantle, and should therefore have the form of cavity plumes. According to [26], the thickness of the dense less viscous layer from which they come must be less than \( 200–700 \) km. The scaling laws of Le Bars and Davaille (2004a, 2004b) predict velocities \( \sim 5–20 \) cm yr\(^{-1} \), radius \( \sim 500–1000 \) km, spacing \( \sim 2000–3500 \) km, and cyclicity \( \sim 100–200 \) My for cavity plumes at the base of the mantle. Moreover, heat flux could vary by as much as \( 200\% \) both laterally and spatially on those same time and length scales (Davaille et al., 2003; Namiki and Kurita, 2003; Le Bars and Davaille, 2004b). Note that the spacing predicted being much smaller than the typical size of a mantle ‘box’, several instabilities should be observed in each box.

One fundamental question now remains: how might the large-scale circulation created by plate tectonics influence the different regimes described above, and especially their time dependence? Can cyclic doming be maintained in such a flow? Experiments including plate-scale motions are now needed.

### 7.03.9.2.2 Entrainment and mixing

**Entrainment from an interface.** Olson’s experiments (1984) on two-layer convection established that entrainment at low Reynolds number can indeed occur, and that viscous stresses can substitute for inertial instabilities as a mixing mechanism. The entrainment occurs in two steps: first, the thermal heterogeneities at the interface induce circulations in the two layers; then the viscous drag due to those convective motions becomes sufficient to overcome the negative buoyancy forces due to the stable chemical density gradient, and thin tendrils of material are entrained (Figure 51(a); Sleep, 1988; Lister, 1989). The entrainment rate and the filament thickness depend on the intensity of convection, its geometry, the viscosity ratio, and the buoyancy ratio: the more stable the fluid, the harder it is to entrain. A significant entrainment corresponds also to a cusp height greater than a critical value comparable to the thickness of the thermal boundary layer above the interface (Jellinek and Manga, 2004). Scaling laws have been derived (Davaille 1999a, 1999b; Davaille...
et al., 2002; Jellinek and Manga, 2002, 2004) which explain the data well (e.g., Figure 51(c)).

Stirring. The mixing pattern in thermochemical convection (Figure 52) is very complex, and still poorly characterized since convection creates compositional heterogeneities with two different typical sizes and topologies: (1) thin filaments generated by mechanical entrainment across the interface, leading to the development of ‘marble cake’ structures in both reservoirs, even though they can remain dynamically separated for a long time and (2) domes, and blobs encapsulated within them, are generated by instabilities.

Experimental limitation: species diffusion. The geodynamical irrelevance of surface tension effects requires the use of miscible fluids in the laboratory. Therefore, chemical diffusion becomes a problem when the ratio of chemical to thermal diffusivity (the Lewis number) \( Le = D/\kappa \) is too small. In the mantle, this ratio is \( >10^9 \). In the laboratory experiments reported here, it was always greater than 1000. Even then, chemical diffusion becomes important when convection is slow (cf. Section 7.03.9.1), and the mechanism of entrainment across the interface then switches to advection-enhanced diffusion (see the discussion in Davaille (1999a), and gray area in Figure 51(b)). Another problem arises when three-component solutions are used (e.g., water + salt + Natrosol) in which the two dissolved species (e.g., salt and Natrosol) do not diffuse at the same speed. Then, if the more viscous layer is initially the upper layer (i.e., the layer with no salt is
the layer with the most Natrosol), salt-Natrosol fingers develop, even before the onset of thermal convection. This phenomenon, well known in thermo-haline convection, changes drastically the physics and the entrainment rate at the interface. This led Davaille (1999a, 1999b) to discard those experiments. This could also explain the results of Namiki (2003), who observed, using Natrosol solutions, that the interfacial entrainment rate when the more viscous layer was on the top was different than when it was at the bottom.

7.03.9.3 Interaction of a Plume with a Density and/or Viscosity Interface

This situation is of interest for the mantle since the 660-km depth seismic discontinuity corresponds to the phase transition from the spinel phase at low pressure to perovskite at higher pressure (see Chapter 2.06). Although this endothermic phase transition probably cannot stratified mantle convection, it is sufficient to delay flow (see Chapters 7.02 and 7.05). Moreover, the perovskite phase is also 3–100 times stiffer than the upper mantle phases.

Laboratory experiments coupled to numerical modelling have shown that a viscosity jump alone is sufficient to modify the shape of an upwelling compositional plume, the head of which becomes elongated as it crosses the discontinuity from the stiffer lower mantle to the less viscous upper mantle (Manga et al., 1993). For sufficiently large viscosity contrast, the plume head can even accelerate so much in the upper mantle that it becomes disconnected from the plume tail, which aggregates at the interface and forms a second plume 'head' (Bercovici and Mahoney, 1994). If the plume is compositionally heterogeneous, segregation can occur at the interface and the two head events then have different geochemical signatures (Kumagai and Kurita, 2000). These phenomena would explain well the emplacement timing and the geochemical data at Ontong Java Plateau (Bercovici and Mahoney, 1994; Kumagai and Kurita, 2000).

Kumagai et al. (2007) extended the previous studies to thermal plumes and investigated the interaction between a lower-layer thermal plume and an inner interface. The interaction mode depends on the local buoyancy number ($B_L$: the ratio of the stabilizing chemical buoyancy to the plume thermal buoyancy at the interface), $Ra$, and $\gamma$. For $B_L < 0.6$, the ‘pass-through’ mode develops, whereby a large volume of the lower material rises through the upper layer and reaches the top surface, since the plume head has a large thermal buoyancy compared to the stabilizing density contrast between the two layers. When $B_L > 0.6$, the ‘rebirth’ mode occurs, where the thermal plume ponds and spreads under the chemical boundary and secondary thermal plumes are generated from the interface. Depending on the magnitude of $B_L$, these plumes can entrain a significant amount of lower-layer material upwards by viscous coupling.

### 7.03.10 Mid-Ocean Ridges and Wax Tectonics

The study of ridges and accretion is challenging because it involves processes which involves both brittle fracture and ductile flow, which is quite difficult to study numerically. For example, current numerical simulations still cannot reproduce the coupled fluid–solid deformation processes responsible for microplates at mid-ocean ridges. On the other hand, published results from a wax analog model yielded the first observations of overlapping spreading centers (OSCs), morphological precursors to microplates, before their discovery on the seafloor (Oldenburg and Brune, 1972). Two main questions have been tackled with laboratory experiments: the ability of a buoyant mantle upwelling to break the crust, and the morphology of ridges.

#### 7.03.10.1 Can a Buoyant Mantle Upwelling Generate Sufficient Stress to Rupture a Brittle Crust?

Using surface layers of various brittle and visco-elastic materials, Ramberg and Sjostrom (1973) studied whether a relatively stiff crustal layer could break in tension above a buoyant diapir and become displaced laterally in a manner simulating the breakup of Pangea and the wandering continents. For better visualization of the deformation, the material was initially dyed in different colored layers. The model was then centrifuged for several minutes at accelerations up to 3000g. All experiments show the breakup of the brittle layer by the diapir head. However, the material rheology was not sufficiently well characterized to permit a quantitative study, or to investigate the morphology of the ridges.

#### 7.03.10.2 Morphology of Ridges

Ridge morphology has been studied using paraffin wax as an analog mantle material: solid wax simulates brittle mantle and molten material simulates the
region that deforms as a viscous fluid (Oldenburg and Brune, 1972, 1975; Ragnarsson et al., 1996). The molten wax was frozen at the surface by a flow of cold air. Then the solid crust was pulled apart with constant velocity and a rift was formed separating the crust into two solid plates (Figure 53). In this case, the solid wax thickness increases as the square root of the distance to the ridge (Oldenburg and Brune, 1975), in agreement with theory (Parker and Oldenburg, 1973).

Oldenburg and Brune (1972) first observed that a straight rift initially perpendicular to the pulling direction evolved into a pattern consisting of straight segments interrupted by faults orthogonal to the rift and parallel to the pulling direction (Figure 54). The ability of a wax to generate orthogonal transform faults depends on the ratio of the shear strength of its solid phase to the resistive stresses acting along the transform fault: if the latter exceed the former, a breakup of the solid near the transform fault wall should occur (Oldenburg and Brune, 1975).

Ragnarsson et al. (1996) and Bodenschatz et al. (1997) studied the phase diagram in more detail, investigating the rift structure as a function of the pulling speed for two different waxes. The regime diagram was found to depend critically on the rheology of the wax:

1. In the first case (Shell Wax 120), the solid layer could be divided in two regions because the paraffin wax undergoes a solid–solid phase transition: a colder phase, hard and brittle, where strain is mostly accommodated by crack formation during extension; and a warmer ductile phase, where strain is accommodated primarily by viscous flow. This wax gave birth to the different regimes shown in Figure 55 (Ragnarsson et al., 1996), but the orthogonal transform faults observed by Oldenburg and Brune (1972, 1975) were never observed. At slow spreading rates, the rift, initially perpendicular to the spreading direction, was stable (Figure 55(a)). Above a critical spreading rate, a ‘spiky’ rift developed with fracture zones almost parallel to the spreading direction (Figure 55(b)). At yet higher spreading rates a second transition from the spiky rift to a zigzag pattern occurred (Figure 55(c)). With further increase of the spreading rate the zigzags steepened (Figure 55(d)). In Figure 55(a) and 55(b), the rift was frozen, whereas in Figures 55(c) and 55(d), molten fluid reached the surface. In the zigzag regime, the angle of the interface with respect to the spreading direction can be described by a simple geometrical model, since the problem is dominated by only two velocities, the externally given spreading rate (pulling speed) and the internally selected maximal growth.

Figure 53  Experimental setup. The tank is heated from below at constant temperature and cooled from above by a constant flow of air. Before each run, a layer of wax is allowed to grow on the surface until thermal equilibrium is reached. Two skimmers embedded in the solid wax are attached to a threaded rod which is driven by a stepping motor. The rift is initiated with a straight cut through the wax, perpendicular to the spreading direction. Divergence at this cut causes liquid wax to rise into the rift and solidify. Illumination from below permits to image the plate thickness at the rift from above using a video camera. From Ragnarsson R, Ford JL, Santangelo CD, and Bodenschatz E (1996) Riffs in spreading wax layers. Physical Review Letters 76: 3456–9.

Figure 54  Transmission image of transform faults, obtained in Shell Wax Callista 158 and with a spreading rate of 78 mm s⁻¹. Image size is 9 mm. From Bodenschatz E, Gemelke N, Carr J, and Ragnarsson R (1997) Riffs in spreading wax layers: An analogy to the mid-ocean rift formation. Localization Phenomena and Dynamics of Brittle and Granular Systems. Columbia University.
velocity of the solidifying fronts. When the spreading rate exceeds the solidification speed, the interface has to grow at an angle to keep up. The experimental data on the angles agree well with this model.

2. In the second case (Shell Wax Callista 158), the solid phase is more brittle (Bodenschtaz et al., 1997; Oblath et al., 2004). Three distinct morphological regimes were observed. At slow spreading rates, a straight rift is stable and forms a topographic low. At moderate rates, it becomes unstable and OSCs and microplates form, evolve, and die on the ridge, which has little or no relief. At high spreading rates, the microplates lose their internal rigidity and become transform faults at the ridge (Figure 54), as previously described by Oldenburg and Brune (1972, 1975). The ridge corresponds to a topographic high (Bodenschtaz et al., 1997; Oblath et al., 2004). Katz et al. (2005) focused on the formation of microplates (Figure 56). In wax, like on Earth, they originate from OSCs. The latter nucleate predominantly on sections of obliquely spreading rift, where the rift normal is about 45° from the spreading direction.

These experiments are very interesting for pattern formation, and show striking geometrical similarities with what is observed on the ocean floor. However, their relevance to mid-ocean ridge systems is still questionable for two reasons: first, dynamic similarity does not obtain, since the viscosity contrast between the solid and the liquid wax is much greater than the viscosity contrast across the solid lithosphere. Experiments involving wax are therefore probably more relevant to the dynamics of ice satellites (e.g., Manga and Sinton, 2004) or of lava lakes. Second, the physics of the wax experiments reported above is far from being understood. For example, the influence of latent heat effects on the dynamics of the wax systems remains unknown. To apply their results to the Earth mid-ocean ridges system, Oldenburg and Brune (1972, 1975) neglected these effects and proposed a semi-quantitative theory based on shrinkage and the mechanical properties of wax. The more recent experiments, although they very nicely showed the diversity of rift morphologies, were not able to relate them quantitatively to the wax cooling, nor to its rheology. So a quantitative...
understanding of lithosphere rifting remains to be achieved, and will require a better knowledge of fluid rheology.

7.03.11 Subduction-Related Experiments

The subduction of oceanic lithosphere is the most prominent signature of thermal convection in the Earth's mantle. Subducting plates constitute the cold downwellings of mantle convection, which sink into the mantle because of their negative buoyancy. Subduction involves a wide variety of physical and chemical processes: earthquakes, volcanism (dehydration, melting, and melting migration), phase transformations, thermal effects, mantle circulation, and plate motion. They are reviewed in detail by King in Chapter 7.08. We focus here on the experimental work which has been restricted to the study of the mantle-scale dynamics of subduction, namely the relations between motion of subducted slabs, mantle flow, and back-arc spreading (Figure 57). Laboratory models of these phenomena have three advantages: they are inherently 3-D; they do not suffer from limited computational resolution of the temperature and velocity fields; and it is easy to work with strong viscosity variations.

7.03.11.1 Ingredients for Subduction

It has been long recognized that cold plates are not produced by isoviscous convection, and that the strong temperature dependence of mantle viscosity is required (e.g., Torrance and Turcotte, 1971), as we showed in Sections 7.03.5 and 7.03.6. Turner (1973) illustrated this idea using glycerine in a tank cooled from above, in which convective overturn was forced by a stream of bubbles of cold CO$_2$ released from dry ice at the bottom of the tank. Surface cooling then produced a thin viscous sheet which was driven away from regions of upward convection, and then plunged steeply into the less viscous interior, maintaining its identity as it did so. Jacoby (1970, 1973, 1976) and Jacoby and Schmeling (1982) studied the gravitational sinking of cold viscous high-density lithosphere into asthenosphere. Jacoby first studied the mode of sinking of a heavy elastic rubber sheet into water under isothermal conditions without active 'mantle' convection (Jacoby, 1973). Later, he used the same paraffin as Oldenburg and Brune (1975) to include the effects of thermal convection. The paraffin was heated from below above its melting point, and cooled from above below its melting point. It therefore formed a thin solid dense skin on top of the fluid layer. However, the skin could not be generated or melted at the same rate as that of the sinking, preventing real plate tectonics behavior (Jacoby, 1976). Moreover, surface tension prevented thick skins from sinking by themselves. However, sinking could be initiated by loading the edge of the skin or by wetting its surface with molten paraffin (Figure 58). The heavy skin first bends with a radius of curvature which depends on the skin thickness, then accelerates down the trench as the portion within the mantle increases. Upon interacting with the tank bottom, the skin can 'fold' and the trench can even 'roll back' in the oceanward direction (Figure 58). Convection cells caused by localized heating at the bottom of the tank do not perturb significantly the slab's evolution. Although these pioneering experiments were only 'semi-quantitative' (Jacoby, 1976), they already reproduced most features of subduction (down-dip motion and roll back, 3-D, time dependence, folding on the box bottom, etc.), as well as highlighting the difficulties that quantitative experiments have to overcome (subduction initiation, surface tension effects, etc.).

Kincaid and Olson (1987) designed experiments to further study the lateral migration of slabs and their penetration through the transition zone (Figure 4). Cold, negatively buoyant molded slabs of concentrated sucrose solution were introduced into a more dilute, two-layer sucrose solution representing the upper and lower mantle. The transition zone was modeled by a step increase in both density and viscosity. The initial setup consisted of two horizontal
plates separated by a trench gap, with one plate attached to a dipping slab, simulating a developing subduction zone with an overriding plate (Figure 57). The trailing end (‘ridge’) of the slab was either locked ($V_P = 0$), or free to move along with the plate. Besides the Prandtl number [2], important dimensionless numbers are the ratio of slab thickness to mantle depth $C_{17}/C_{14}$ and the ratios $S_{\eta_S}/U_{\eta_U}$ and $S_{\eta_S}/L_{\eta_L}$ of the slab viscosity to those of the upper and lower mantle. It was necessary to have $S_{\eta_S}/U_{\eta_U} > 10^3$ to maintain a tabular shape during subduction. No velocity is imposed and the slab sinks under its own weight. This phenomenon can be described by an effective Rayleigh number, defined in terms of the slab/upper layer density contrast $\rho_S - \rho_U$:

$$Ra_S = \frac{(\rho_S - \rho_U)gH^3}{\kappa\eta_U}$$  \[28\]

In the experiments, $Ra_S \sim 1.2 \times 10^5$. On the other hand, the style of slab penetration through the density discontinuity depends on the ratio of the slab buoyancy in the upper and the lower mantle:

$$B_S = \frac{S_{\rho_S} - L_{\rho_L}}{S_{\rho_S} - U_{\rho_U}}$$  \[29\]

Homogeneous fluids correspond to $B_S = 1.0$, while negative values correspond to strong stratification. In agreement with the numerical study by Christensen and Yuen (1984), Kincaid and Olson (1987) found three modes of slab penetration (Figure 59(a)). Regime I, in which $B_S \leq -0.2$, corresponds to strong stratification and virtually no slab penetration. In the intermediate regime II ($-0.2 \leq B_S \leq 0.5$), there is partial slab penetration and formation of a root beneath the interface. When $B_S > 0.5$, the stratification is weak enough to permit the slab to sink into the lower layer with minor deformation (regime III). Retrograde slab motion and trench migration occurred in nearly every case, being greatest in regime I (Figure 59(b)), episodic in regime II, and nearly absent in regime III, especially when $L_{\eta_L}/U_{\eta_U}$ is around 1. Moreover, the episodicity of trench rollback is related to the interaction of the slab with the density stratification, and is greatest in the case of a fixed ridge. The complexity of the time-dependent slab motion, both downdip and retrograde, and its interaction with any bottom boundary, made it difficult to derive scaling laws to describe subduction. Work in the last 20 years has aimed at obtaining those scalings, using experiments where more parameters were controlled.

### 7.03.11.2 The Surface Story and the Initiation of Subduction

Subduction initiates within the lithosphere and produces large-scale tectonic features. The first analog modeling of tectonic processes probably dates back to 1815 when Sir James Hall attempted to model folds in geological strata, using layered beds of clay and clothes. Numerous experiments have been performed since to reproduce lithosphere and crustal dynamics. It is beyond the scope of this chapter to review them all (for recent reviews, see Schellart (2002)). We shall describe here only studies relevant to the nucleation and evolution of subduction.

Lithospheric processes necessitate taking into account the complex rheology of the lithosphere. Shemenda (1992, 1993) considered the subduction...
of an elasto-plastic lithosphere (a mixture of powder and hydrocarbons) into a low-viscosity asthenosphere (water), driven by both horizontal gravitational sinking (i.e., plate/asthenosphere density contrast) and an horizontal compressional force driven by a piston. However, subduction did not initiate without the presence of pre-existing discontinuities. In this case, the horizontal compression produced buckling instabilities, then localization of deformation, leading to failure and eventually subduction of the lithosphere. Trench rollback was then accompanied by back-arc opening along pre-existing faults. The lithosphere has also been modeled as a succession of brittle and ductile layers of different strengths and rheologies (e.g., Davy and Cobbold, 1991). Following this approach, Pinet and Cobbold (1992) and Pubellier and Cobbold (1996) studied the consequences of oblique subduction, that is, the partitioning between down-dip motion and transverse motion along faults. They used a sand mixture to model the brittle upper crust, silicone putty for the ductile lower crust/upper mantle, and glucose syrup for the asthenosphere.

Faccenna et al. (1996, 1999) and Becker et al. (1999) use the same layered system to study the behavior of an ocean-continent plate system subjected to compressional strain over geological timescales. Compressional stress was achieved by displacing a piston at constant velocity perpendicular to the plate margin (Figure 60). The coupled interface between oceanic and continental plates is found to resist a rapid surge of compressional stress and the shortening is accommodated by undulations within the ocean plate (Martinod and Davy, 1994). But if the system evolves under a low compressive strain rate (slow ridge-push), the oceanic plate becomes unstable to

![Figure 59](a) Styles of slab penetration through a density discontinuity: I, slab deflection with $B_s < 0.2$; II, partial slab penetration and formation of a root beneath the interface with $B_s < 0$; III, complete slab penetration for $B_s > 0.5$. (b) Trench migration and retrograde subduction with the fixed ridge boundary condition. The dye streak indicates the flow pattern in the back arc edge. From Kincaid C and Olson P (1987) An experimental study of subduction and slab migration. Journal of Geophysical Research 92: 13832–13840.

![Figure 60](Four layers setup to study the initiation of subduction. From Faccenna C, Giardini D, Davy P, and Argentieri A (1999) Initiation of subduction at Atlantic-type margins: Insights from laboratory experiments. Journal of Geophysical Research 104: 2749–2766.)
RTI and subduction develops (Faccenna et al. 1999). The trench is then first localized at the ocean-continent boundary because of the lateral heterogeneity there. The dynamics of the system can therefore be controlled by the ‘buoyancy number’ $F$, the ratio of driving buoyant to resisting viscous forces (e.g., Houseman and Gubbins, 1997):

$$F = \frac{\rho_s - \rho_L}{\eta_S \cdot H} = \frac{\rho_s - \rho_L}{\eta_S \cdot U} \frac{g \delta_s}{U}$$

where $U$ is the imposed horizontal velocity, and $\sigma$ is the RTI growth rate for a more viscous slab (see Section 7.03.4). If $F(\rho_S - \rho_L, \delta_S, \eta_S, U) \leq 1$, the RTI is inhibited and the oceanic plate is deformed by folding. Subduction initiates if $F(\delta_S) > 1$, that is, if the negative buoyancy force of the oceanic plate exceeds the viscous resisting force of the model lithosphere. Faccenna et al. (1999) further show that the passive margin behavior is not sensitive to the high shear strength of the brittle layer but only to the resistance of the ductile layer. So subduction initiation is essentially a viscous fluid process. Since its growth rate $\sigma$ depends on the inverse of the slab viscosity (which is high), it is a slow process. However, once the developed slab sinks into the less viscous mantle, its style and dynamics are predominantly governed by a buoyancy number $F(\rho_S - \rho_L, H, \eta_S, U)$ based on the viscosity and the thickness of the upper mantle (Becker et al., 1999). The velocity and angle of subduction and the rate of trench rollback are found to be strongly time dependent and to increase exponentially over tens of millions of years before the slab interacts with the 660 km discontinuity.

7.03.11.3.1 Generation of seismic anisotropy

Motivated by the growing body of evidence that trench rollback could be associated with a particular pattern of seismic anisotropy (see Chapters 1.16 and 2.16 volume 2), whereby the $a$-axis of olivine crystals would align parallel to the trench oceanward of the slab and turn around its edges (e.g., Russo and Silver, 1994), Butter and Olson (1998) used small cylinders (whiskers) suspended in a viscous fluid as an analog to olivine crystals, and studied their orientation in vertical and horizontal cross-sections in the vicinity of a subducting slab. The slab was simulated by a rigid Plexiglas plate whose dip angle, down-dip motion, and rollback were controlled independently (Figure 60), and the mantle was modeled with Newtonian corn syrup. The whole system was isothermal. They show that trench-parallel olivine $a$-axis orientation in the seaward-side mantle is indeed controlled by the amount of slab rollback, and that orientations in the mantle wedge depend upon slab dip angle.

7.03.11.3.2 Mantle flow and thermal evolution of the slab

Kincaid and Griffiths (2003, 2004) used the same kind of setup to study the variability of flow and temperature in subduction zones. The slab was made of a composite laminate cooled to a prescribed temperature before the experiment, and containing temperature sensors to monitor its thermal evolution as it is forced into an isothermal tank of hotter glucose syrup. The thermal boundary layers developing in the fluid were visualized by a Moiré technique and the flow field measured using tiny air bubbles and PIV (Figure 61). The experiments $Pe$ ranged between 80 and 450. The mantle return flow induced by pure longitudinal sinking creates two cells, one oceanward and one in the wedge (Figure 61), where the velocity can reach 40% of the slab downdip speed. Rollback subduction induces flow both around and beneath the sinking slab (Figure 25), with larger velocities in the wedge and flow focused toward the center of the plate. These 3-D flows strongly influence the thermal evolution and structure of the plate: they speed up slab reheating. Moreover, the highest slab reheating occurs along the slab centerline when there is rollback, while it occurs along the edges for pure longitudinal sinking.
These experiments consider the interactions between the mantle and a mature, purely viscous slab. The subduction is induced either by extruding a viscous slab with constant velocity and dip angle (e.g., Figure 61), or by bending the tip of an horizontal slab and forcing it into the mantle until the slab sinks under its own weight (e.g., Figure 57).

### 7.03.11.4 Fixed rollback velocity

These experiments were designed to further investigate the interaction of a sinking slab with a density and viscosity interface, mimicking the discontinuity at 660 km depth between the upper and lower mantles (Kincaid and Olson, 1987) (Figures 62 and 63). The slab velocity and initial dip angle were this time imposed. Griffiths and Turner (1988) injected tabular slabs and cylindrical plumes vertically ($V_T = 0$) onto a fluid interface under isothermal conditions and determined the conditions under which folding of the slab or plume occurred. Mixtures of water and Golden Syrup were used to control the viscosity and density of the slab and the mantle layers. As in the experiments of folding on a solid surface (Cruickshank and Munson, 1981; Figure 64(a)), they found that folding occurs in a compressed slab when the ratio of its length to its width exceeds a critical value ($\sim 6$). They also determined the amount of fluid entrained within the folds in the lower layer, and when this entrainment could sufficiently reduce the buoyancy of the slab to prevent its sinking further in the lower mantle. Griffiths et al. (1995) and Guillou-Frottier et al. (1995) extended these results to retreating slabs ($V_E$ and $V_T$ constant). Griffiths et al. (1995) used again isothermal mixtures of Golden Syrup and water so that their slab was between 1.5 and 50 times more viscous than their lower mantle. Guillou-Frottier et al.
(1995) used corn syrup, and extruded a very cold and viscous slab into their two-layer mantle, so that the experiments $Pe \sim 30–150$ and the slab was $10^2–10^3$ more viscous than the lower mantle. Both sets of experiments highlighted the essential role of trench rollback, in addition to $B_S$ and $\eta_L/\eta_U$, in the slab penetration regime. Increasing trench rollback enhances slab flattening on the interface (Figures 59 and 65). Increased resistance in the lower mantle also promotes folds and piles (Figure 64(b)), which can either stagnate in the lower mantle or sink to the bottom (Guillou-Frottier et al., 1995). The amplitude of the observed folds is well predicted by numerically determined scaling laws, which also predict amplitudes (400–500 km) consistent with tomographic images of slabs beneath some subduction zones (Ribe et al., 2007). On the other hand, when the slab is not too viscous, RTTs have time to develop from the slab resting on the interface, and to sink into the lower mantle (Figure 65; Griffiths et al., 1995). When, however, the slab material reaches the depth of neutral buoyancy, it will spread at this level as a gravity current (Kerr and Lister, 1987; Lister and Kerr, 1989). Slab deformation modes therefore finally depend on three parameters: the slab to mantle viscosity ratio $\eta_S/\eta_L$, the ratio of sinking (Stokes) velocities in the upper and lower mantle ($\sim B_S\eta_U/\eta_L$), and the ratio of the slab’s horizontal velocity to its vertical velocity. As the latter can change through time in Nature, a slab probably passes through a number of different deformation modes during its history.

### 7.03.11.4.2 Free rollback velocity

If rollback controls in part the slab deformation at depth, what happens if the trench is free to move in response to the mantle flow? A number of systematic studies have recently been devoted to the dynamics of a slab sinking under its own weight (Figure 66(a)). Regardless of the boundary conditions, subduction is always strongly time dependent. After initiation, subduction accelerates as the mass of slab in the mantle increases (slab-pull) while the slab dip is close to 90°. Then, subduction slows down after interaction with the 660 km interface, reaching a steady state when the slab spreads on the interface, followed by eventual penetration, accompanied by folding (Becker et al., 1999; Funiciello et al., 2003, 2004, 2006; Schellart, 2004a, 2004b, 2005). Besides, the amount...
of trench and slab rollback depends on the degree of lateral confinement of the flow (Funiciello et al., 2003, 2004; Schellart, 2004a, 2005), and is much reduced in 2-D or when the side walls are within 600 km of the slab edges (Funiciello et al., 2003). As in the experiments with rigid slabs (Buttles and Olson, 1998; Kincaid and Griffiths, 2003, 2004), slab rollback induces flow around the slab edges. The lateral flow in turn forces the hinge line to adopt a convex shape toward the direction of retreat (Funiciello et al., 2003; Schellart, 2004a; Figure 63). By varying the thickness, width, viscosity, and density of the slab and mantle, Bellahsen et al. (2005) observed three characteristic modes of subduction for a slab connected to a plate with a free ridge (Figures 11(b) and 11(d)): a retreating trench mode (mode I), a retreating trench mode following a transient period of advancing trench (mode II), and an advancing trench mode (mode III). The same modes are found when the plate velocity \( V_P \) is fixed at the ridge (‘ridge push’; Funiciello et al., 2004; Schellart, 2005); a relatively low \( V_P \) results in relatively fast hinge-retreat \( V_T \) with backward sinking of the slab and a backward draping slab geometry (mode I). With increasing \( V_P \), hinge migration is relatively small, resulting in subvertical sinking of the slab and a folded slab piling geometry (mode II). For very high \( V_P \), the hinge migrates forward, resulting in a forward draping slab geometry. These three modes are characterized by different partitioning of the subduction into plate and trench motion. The lithospheric radius of curvature, which depends upon plate characteristics (stiffness and thickness) and the mantle thickness, exerts a primary control on the trench behavior. Moreover, the subduction velocity results from the balance between acting and resisting forces, where lithospheric bending represents 75–95% of the total resisting forces (Bellahsen et al., 2006). Martinod et al. (2005) studied how ridges and plateaus on the subducting lithosphere modify the subduction regime. Using feature tracking image analysis, Funiciello et al. (2006) further investigated the influence of plate width and mantle viscosity/density on rollback and the induced poloidal and toroidal components of the mantle circulation. The poloidal component is the response to the viscous coupling between the slab motion and the mantle, while the toroidal component is produced by lateral slab migration. The experiments show that both components are important from the beginning of the subduction process, and strongly intermittent.

The experiments have therefore established that mantle subduction is a strongly time-dependent and 3-D phenomenon. Hence, mantle flow in subduction zones cannot be correctly described by models assuming a 2-D steady-state process. Subduction dynamics involve slab-pull, ridge-push, but also trench rollback and interaction with the 660 km depth discontinuity. All these effects...
explain the diversity of slab deformation and subduction history observed in the laboratory and in Nature (e.g., Kincaid and Olson, 1987; Guillou-Frottier et al., 1995; Griffiths et al., 1995; Faccenna et al., 2001; Schellart, 2004a). Despite the multiplicity of regimes, time-dependent trench rollback is the most common occurrence, which explains well the formation of back-arc basins (e.g., Faccenna et al., 2001). The challenge now is to obtain scaling laws to predict the complete regime diagram of subduction and the characteristics of each regime. This will require including thermal effects, and addressing the problem of lithospheric rheology. All the experiments which have been done so far used a slab with Newtonian rheology. However, the true ‘effective’ viscosity of the lithosphere is still controversial. In the laboratory, it has been chosen either $10^3$–$10^4$ times more viscous than the mantle (e.g., Olson and co-workers, Funicello and co-workers) so that the slab retains its tabular shape, or only 10–100 times more viscous (e.g., Griffiths and co-workers, Schellart), which produced somewhat different results. Another problem is to estimate the influence of surface tension, which is unavoidable in laboratory experiments with free surfaces (Jacoby, 1976) and which must be accounted for to obtain quantitative measurements of the forces resisting subduction and laboratory-based scaling laws for trench motion that can be applied to the mantle.

### 7.03.12 Conclusions

Quantitative analog laboratory experiments have played an indispensable role in advancing the field of geodynamics since the earliest days of its existence as a discipline. Reflecting on this history helps one to understand more clearly what an experimental approach has to offer. At the top of any list must surely be the discovery of new phenomena. Laboratory experiments have always been the source of many of the most exciting new discoveries in fluid mechanics generally, and geodynamics itself offers numerous examples: a partial list would include the convective planforms in a variable-viscosity fluid, the diversity of thermal and compositional plumes, the entrainment in plumes, and the different modes of subduction. A second important contribution of the experimental approach is to the formation of influential new concepts and models. An obvious example here is the classical ‘head plus tail’ picture of a mantle plume; it is no exaggeration to say that this model had its origin almost entirely in the remarkable laboratory experiments of Whitehead and Luther (1975), Olson and Singer (1985), and Griffiths (1986a) among others, and in the beautiful photographic images that came from them. A third important role that laboratory experiments can play is to test theories arrived at using other methods: one thinks for example of the experiments performed by Whitehead and Luther (1975) to test the linear theory of the Rayleigh–Taylor instability. Experimental tests are especially important whenever the theory in question involves assumptions of an asymptotic nature, or whose validity is not obvious.

The rapid evolution of high-performance numerical computing in recent years has provided still another role for experimental geodynamics: that of verifying and benchmarking complex numerical codes for phenomena such as convection with temperature-dependent viscosity (Busse et al., 1994), subduction (Schmeling et al., 2004), and convection with a chemical field (van Keken et al., 1997). Such benchmarks are particularly important for 3-D time-dependent flows with large variations in material properties, which are still difficult to treat reliably by numerical means. By the same token, the ability of numerical models to deliver detailed representations of 2-D and 3-D field variables has encouraged the development of powerful new techniques for quantitative flow visualization in the laboratory, such as the observation of the temperature field using TLCs (e.g., Rhee et al., 1984; Davaille et al., 2006), or the determination of the velocity and stretching fields by PIV (e.g., Adrian, 1991; Voth et al., 2002; Funicello et al., 2006).

While the future of experimental geodynamics is impossible to predict, it is not hard to identify some critical scientific questions where an experimental approach is likely to be fruitful. One is the nature of entrainment and mixing between different mantle reservoirs, which involves phenomena occurring on short length scales and with little diffusion that are hard to resolve numerically. A second is the generation of plate tectonics by mantle convection, which cannot be understood in terms of traditional Newtonian fluid mechanics. Modeling this process in the laboratory will require new developments in the characterization and experimental deployment of fluids with complex rheology.
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